

Deep penetration and total transmission in lossy media

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LABCEM II

Outline

- Inhomogeneous waves at planar interfaces
- Electromagnetic waves penetration in dissipative media
- Deep-penetrating wave
- Deep penetration and Leaky-waves antennas
- Total-transmission in dissipative media
- Conclusions

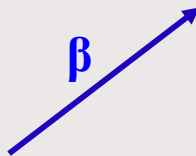
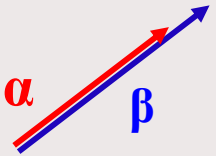
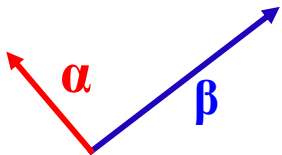
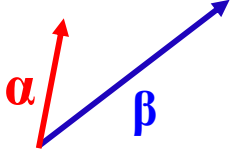
Homogeneous and inhomogeneous waves

An electromagnetic wave propagating in a free space must respect the *dispersion equation*: $\mathbf{k} \cdot \mathbf{k} = k_0^2 \epsilon_r$

Where \mathbf{k} is the propagation vector,

k_0 is the wave number, $(k_0 = \omega \sqrt{\mu_0 \epsilon_0})$

$\epsilon_r = \epsilon_r' - j\epsilon_r''$ is the relative permittivity of the material filling the free space.

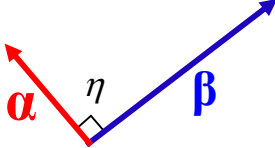
	Lossless Media	Lossy Media
Homogeneous	$\alpha = 0$ 	$\alpha \neq 0$ $\beta \cdot \alpha = \beta \alpha$ 
Inhomogeneous	$\alpha \neq 0$ $\beta \cdot \alpha = 0$ 	$\alpha \neq 0$ $\beta \cdot \alpha = \beta \alpha \cos \eta$ 

Inhomogeneous waves

$$\alpha \neq 0$$

Lossless Media

$$\alpha \neq 0$$

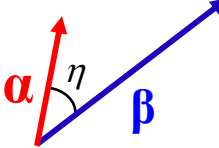
$$\beta \cdot \alpha = 0$$


$$\eta = \frac{\pi}{2}$$

β, α are defined by the antenna

Lossy Media

$$\alpha \neq 0$$

$$\beta \cdot \alpha = \beta \alpha \cos \eta$$


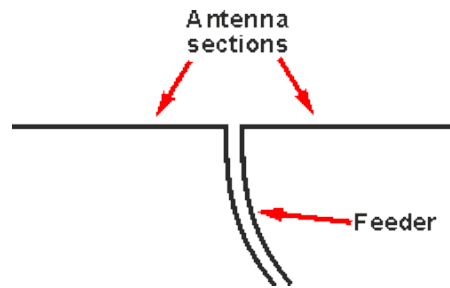
$$|\eta| < \frac{\pi}{2}$$

$$\beta = k_0 \sqrt{\frac{\epsilon'_r}{2}} \sqrt{\sqrt{1 + \left(\frac{\epsilon''_r}{\epsilon'_r \cos \eta}\right)^2} + 1}$$

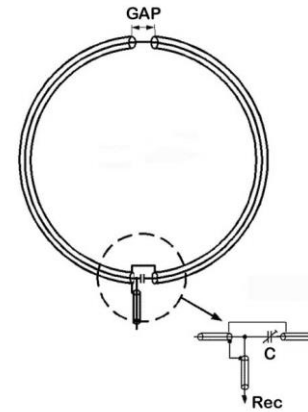
$$\alpha = k_0 \sqrt{\frac{\epsilon'_r}{2}} \sqrt{\sqrt{1 + \left(\frac{\epsilon''_r}{\epsilon'_r \cos \eta}\right)^2} - 1}$$

GPR antennas

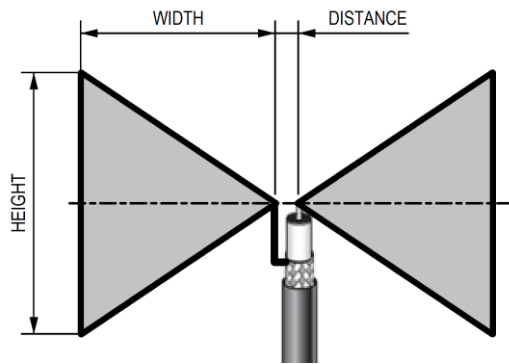
Dipole antenna



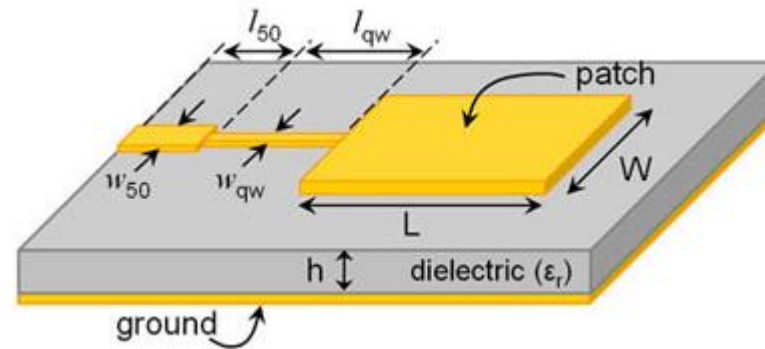
Loop antenna



Bow-tie antenna

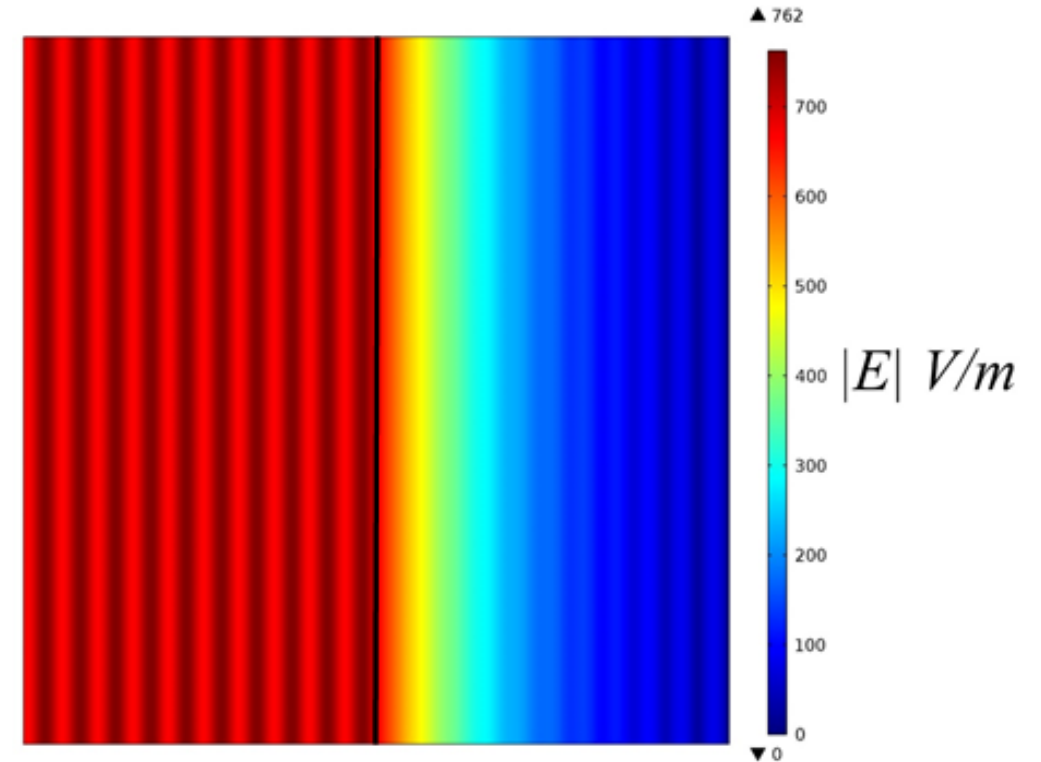
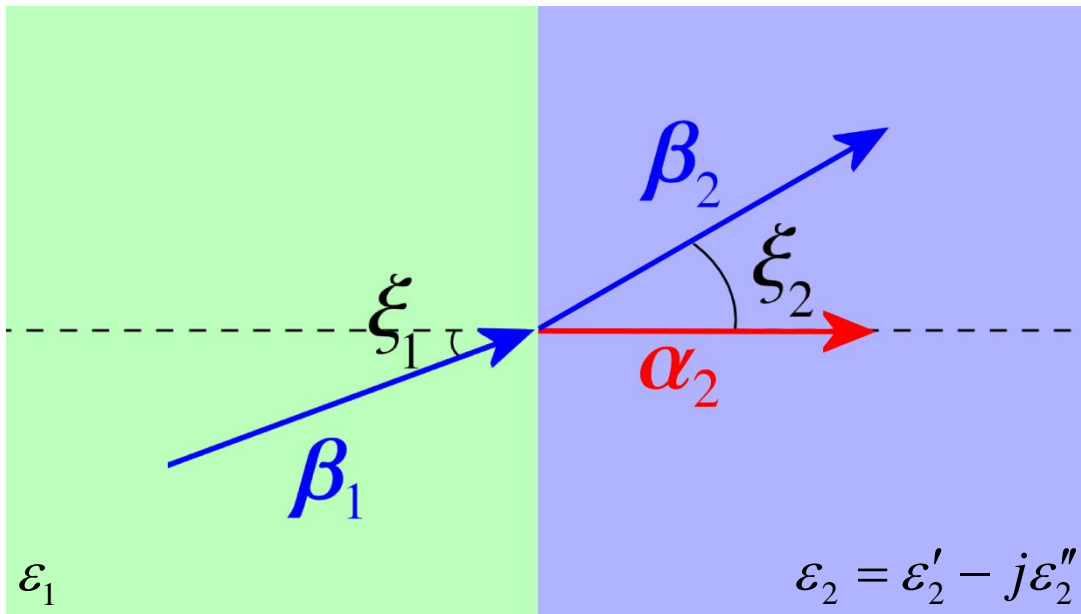


Patch antenna

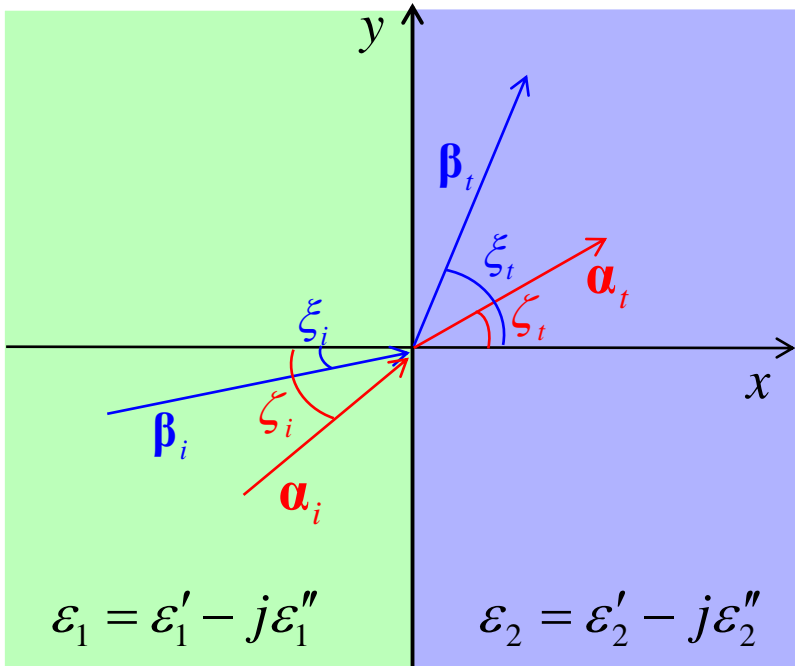


Transmission in dissipative media

As it is well known, when a homogeneous electromagnetic wave, from a lossless medium, impinges on a dissipative material, the transmitted wave is attenuated in the direction perpendicular to the planar interface

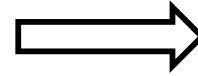


Transmission of inhomogeneous waves



$$\mathbf{k}_i = \boldsymbol{\beta}_i - j\boldsymbol{\alpha}_i$$

$$\mathbf{k}_t = \boldsymbol{\beta}_t - j\boldsymbol{\alpha}_t$$



$$\boldsymbol{\beta}_i = \beta_1 (\mathbf{x}_0 \cos \xi_i + \mathbf{y}_0 \sin \xi_i)$$

$$\boldsymbol{\alpha}_i = \alpha_1 (\mathbf{x}_0 \cos \zeta_i + \mathbf{y}_0 \sin \zeta_i)$$

$$\boldsymbol{\beta}_t = \beta_2 (\mathbf{x}_0 \cos \xi_t + \mathbf{y}_0 \sin \xi_t)$$

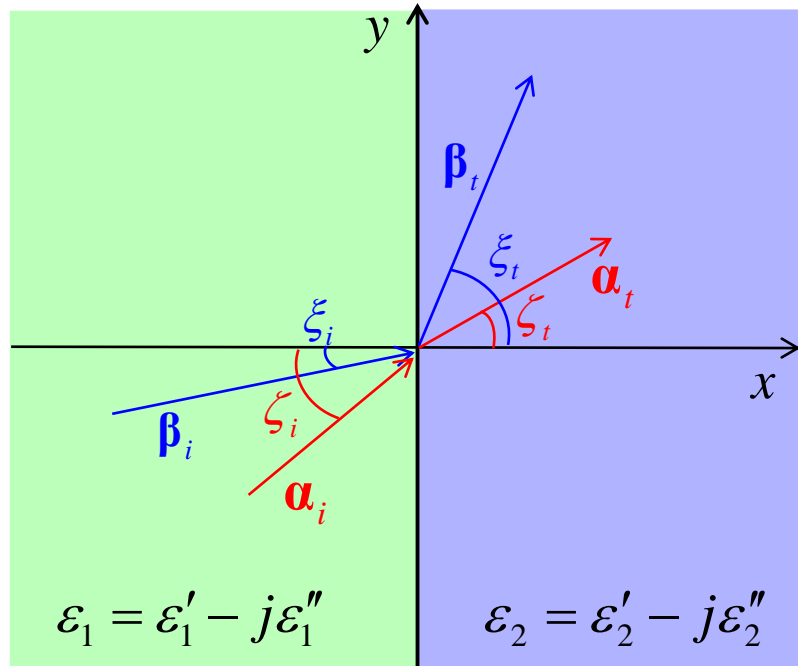
$$\boldsymbol{\alpha}_t = \alpha_2 (\mathbf{x}_0 \cos \zeta_t + \mathbf{y}_0 \sin \zeta_t)$$

How can we find the transmitted wave?

From the continuity of the tangential components!

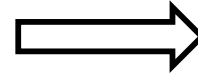
$$\begin{cases} \beta_1 \sin \xi_i = \beta_2 \sin \xi_t \\ \alpha_1 \sin \zeta_i = \alpha_2 \sin \zeta_t \end{cases} \quad \text{Generalized Snell condition}$$

Transmission of inhomogeneous waves



$$\mathbf{k}_i = \boldsymbol{\beta}_i - j\boldsymbol{\alpha}_i$$

$$\mathbf{k}_t = \boldsymbol{\beta}_t - j\boldsymbol{\alpha}_t$$



$$\boldsymbol{\beta}_i = \beta_1 (\mathbf{x}_0 \cos \xi_i + \mathbf{y}_0 \sin \xi_i)$$

$$\boldsymbol{\alpha}_i = \alpha_1 (\mathbf{x}_0 \cos \zeta_i + \mathbf{y}_0 \sin \zeta_i)$$

$$\boldsymbol{\beta}_t = \beta_2 (\mathbf{x}_0 \cos \xi_t + \mathbf{y}_0 \sin \xi_t)$$

$$\boldsymbol{\alpha}_t = \alpha_2 (\mathbf{x}_0 \cos \zeta_t + \mathbf{y}_0 \sin \zeta_t)$$

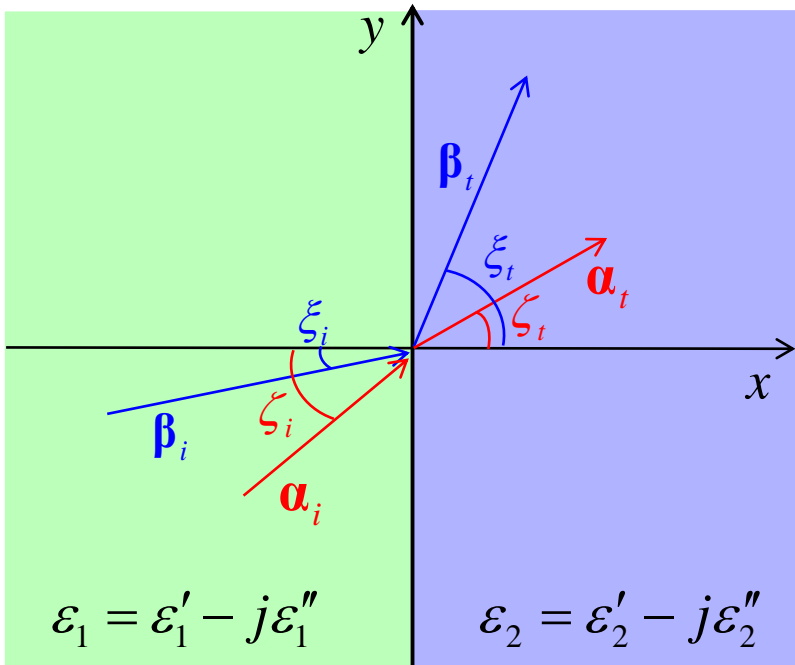
How can we find the transmitted wave?

From the continuity of the tangential components!

$$\begin{cases} \beta_1 \sin \xi_i = \beta_2 \sin \xi_t \\ \alpha_1 \sin \zeta_i = \alpha_2 \sin \zeta_t \end{cases} \quad \text{Generalized Snell condition}$$

Two equations, four unknowns...we are missing something!

Transmission of inhomogeneous waves (2)



Generalized Snell condition

$$\begin{cases} \beta_1 \sin \xi_i = \beta_2 \sin \xi_t \\ \alpha_1 \sin \zeta_i = \alpha_2 \sin \zeta_t \end{cases}$$

We have the dispersion equation:

$$\beta_2^2 - \alpha_2^2 = k_0^2 \varepsilon'_2$$

$$2\beta_2 \alpha_2 \cos(\zeta_2 - \xi_2) = k_0^2 \varepsilon''_2$$

Now, with some algebra, we can get the solution:

$$\beta_2 = \sqrt{\frac{|k_{iy}|^2 + k_0^2 \varepsilon'_2 + |k_{iy}^2 - k_2^2|}{2}}$$

$$\alpha_2 = \sqrt{\frac{|k_{iy}|^2 - k_0^2 \varepsilon'_2 + |k_{iy}^2 - k_2^2|}{2}}$$

Amplitudes of the transmitted vectors

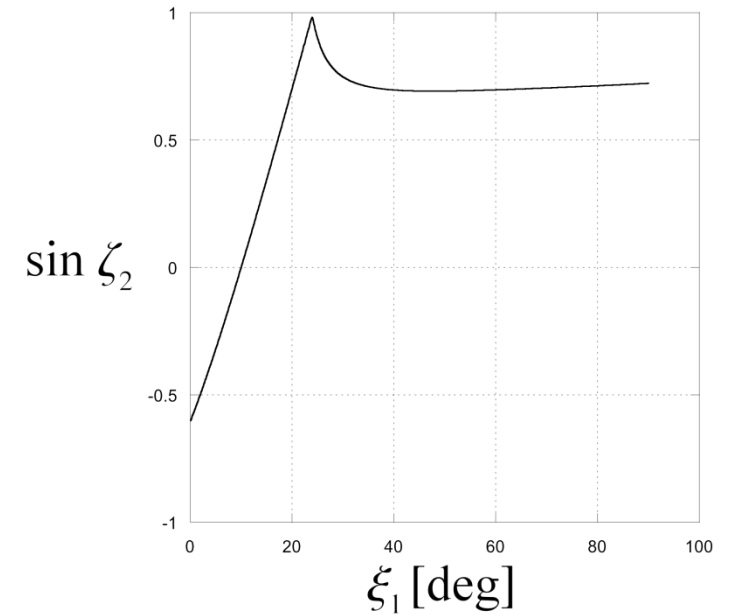
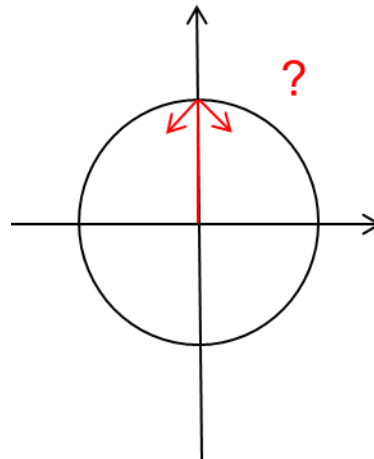
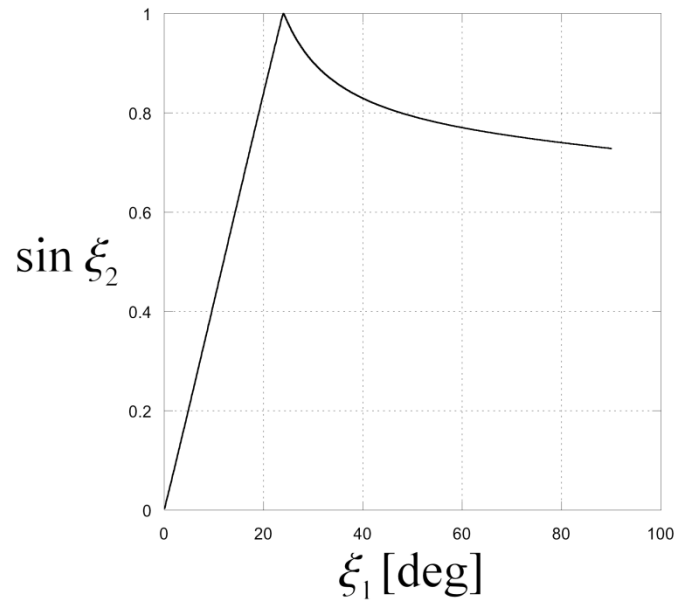
Angles uncertainty

We know the transmitted amplitudes; we need the transmitted angles.

From the generalized Snell condition:

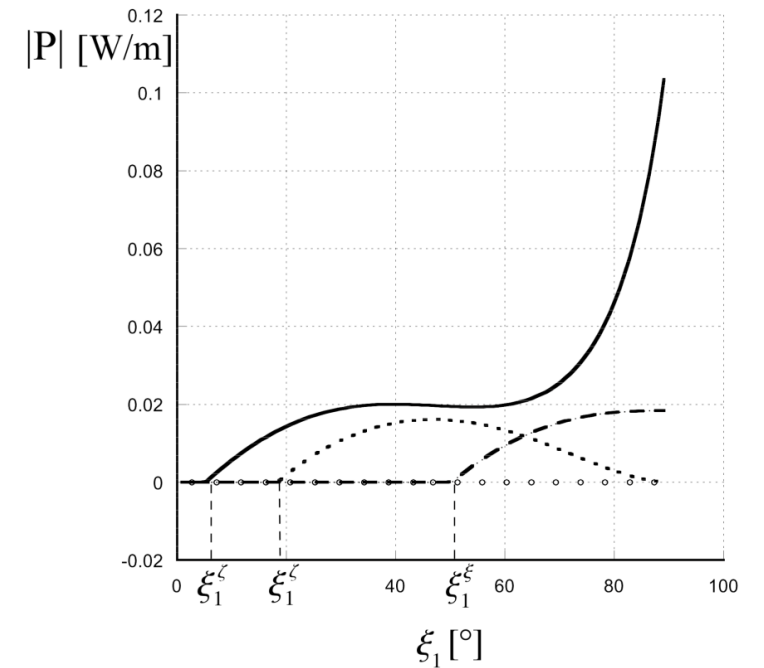
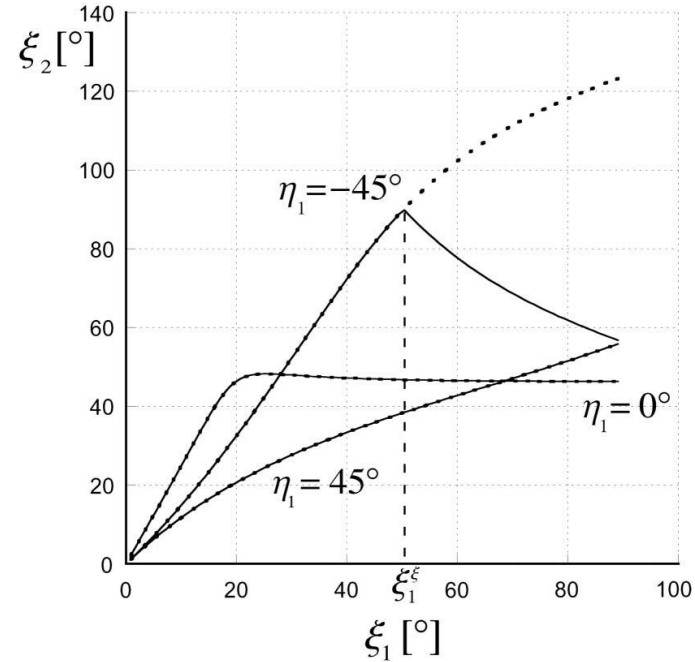
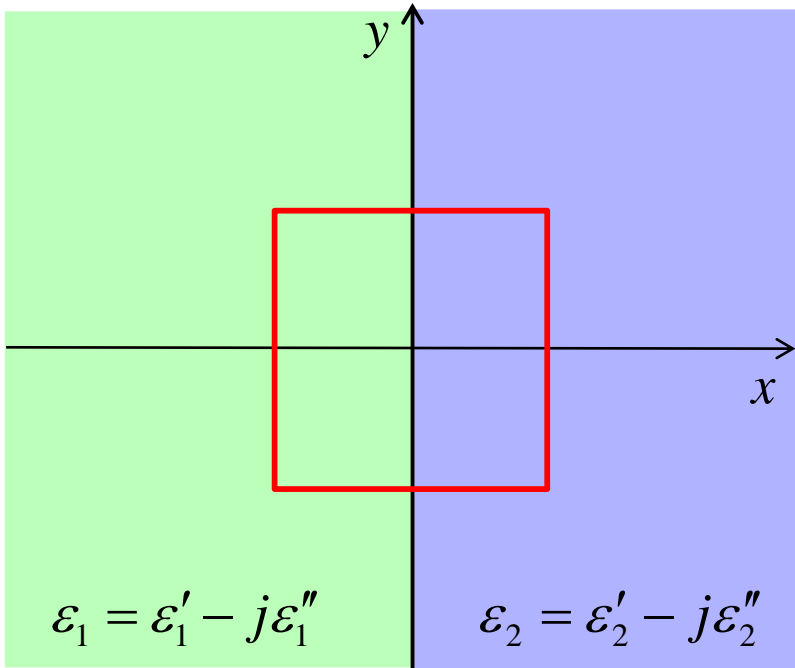
$$\sin \xi_t = \frac{\beta_1}{\beta_2} \sin \xi_i$$

$$\sin \zeta_t = \frac{\alpha_1}{\alpha_2} \sin \zeta_i$$



Angles uncertainty (2)

When we have a doubt, we check the energy conservation!



When the angles come back to 0, the energy does not conserve!

The transmitted angles keep growing after ξ_1^ξ and ξ_1^ζ

Transmitted angles

The, not so simple, expressions of the transmitted angles:

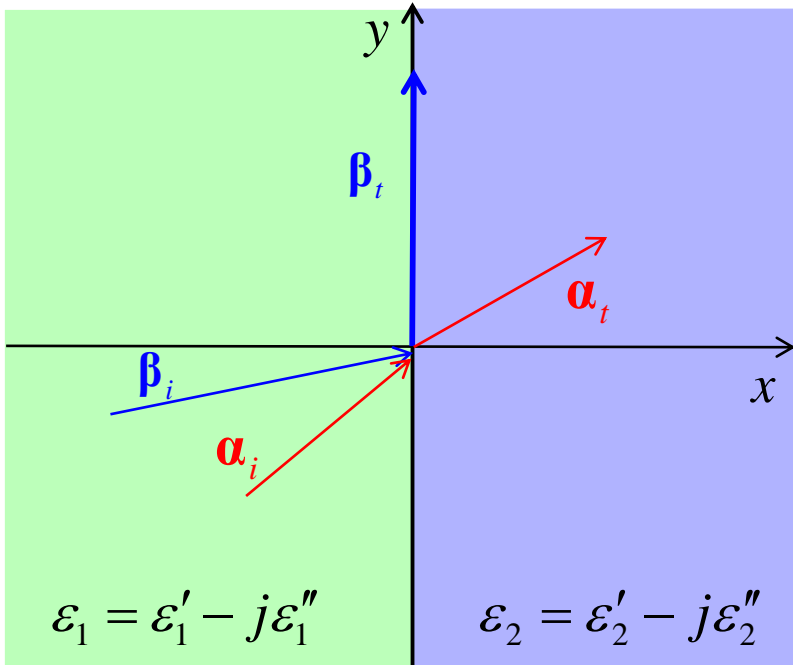
$$\xi_t = \begin{cases} \arcsin\left(\frac{\beta_1}{\beta_2} \sin \xi_i\right) & \text{with } \xi_i \leq \xi_i^{\xi} \\ \pi - \arcsin\left(\frac{\beta_1}{\beta_2} \sin \xi_i\right) & \text{with } \xi_i > \xi_i^{\xi} \end{cases}$$
$$\zeta_t = \begin{cases} \arcsin\left(\frac{\alpha_1}{\alpha_2} \sin \zeta_i\right) & \text{with } \zeta_i \leq \zeta_i^{\zeta} \\ \pi - \arcsin\left(\frac{\alpha_1}{\alpha_2} \sin \zeta_i\right) & \text{with } \zeta_i > \zeta_i^{\zeta} \end{cases}$$

What happens when $\xi_i = \xi_i^{\xi}$ or $\zeta_i = \zeta_i^{\zeta}$?

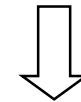
Parallel-phase condition

Let analyse the conditions:

$$\xi_i = \xi_t \implies \sin \xi_t = 1 \implies \xi_t = \frac{\pi}{2}$$



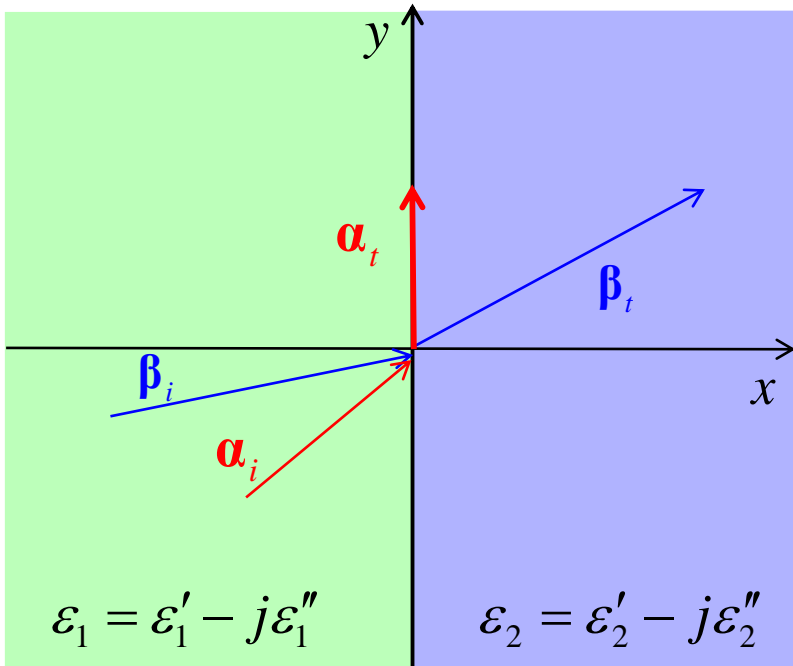
The transmitted wave has
the constant phase plane orthogonal to the interface



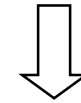
The transmitted wave is similar to a **surface wave**

Parallel-attenuation condition

$$\xi_i = \xi_i^{\zeta} \implies \sin \zeta_t = 1 \implies \zeta_t = \frac{\pi}{2}$$



The transmitted wave has
the constant amplitude plane orthogonal to the interface

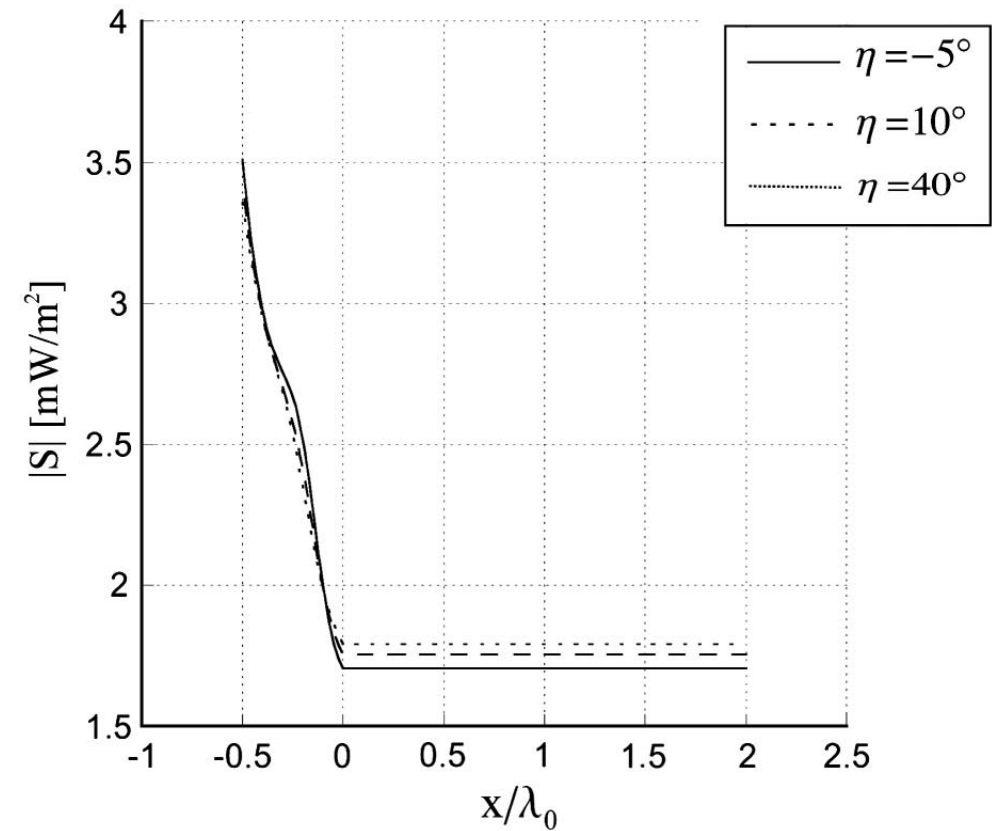
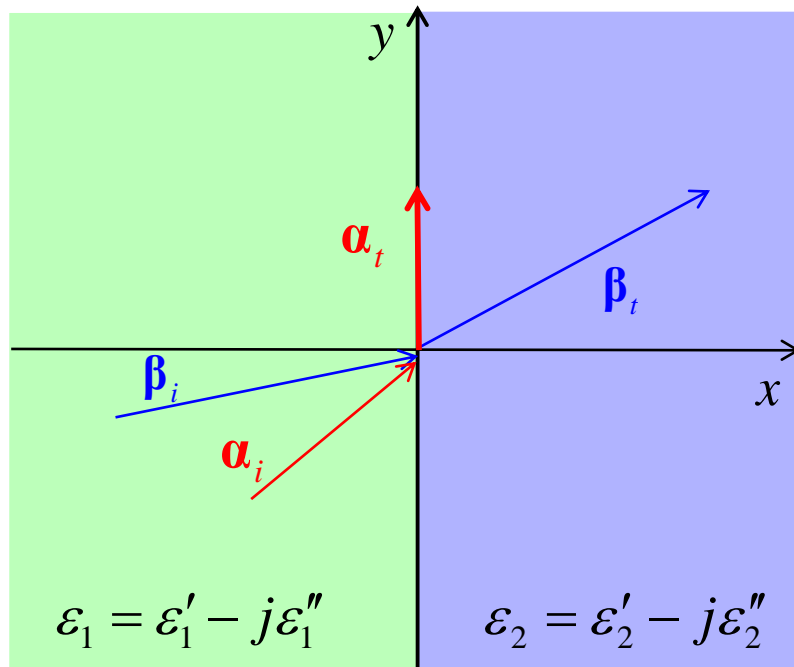


The transmitted wave is not attenuated
towards the second medium!

This is what we call Deep-penetrating wave

Deep-Penetrating Wave (DPW)

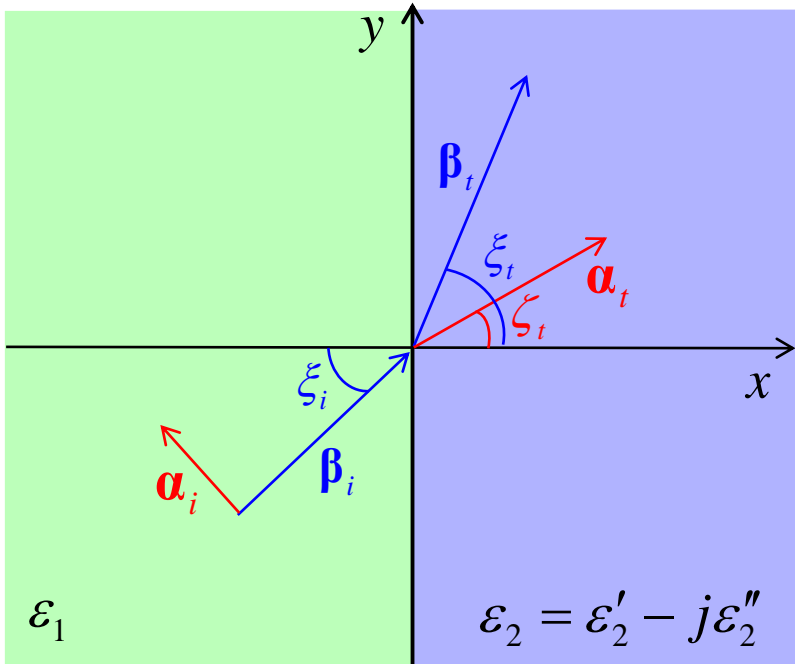
What a strange behavior! Looking at the radiating power as a function of the deepness, it decays in the first medium because the losses, but remains constant in the second medium in spite of losses!



Deep-Penetrating Wave (2)

Let now focusing on the case the first medium is lossless.

As we seen in this case we can still have an incident inhomogeneous wave



Generalized Snell condition in DPW case

$$\begin{cases} \beta_1 \sin \xi_i = \beta_2 \sin \xi_t \\ \alpha_1 \cos \xi_i = \alpha_2 \end{cases}$$

From the dispersion equation:

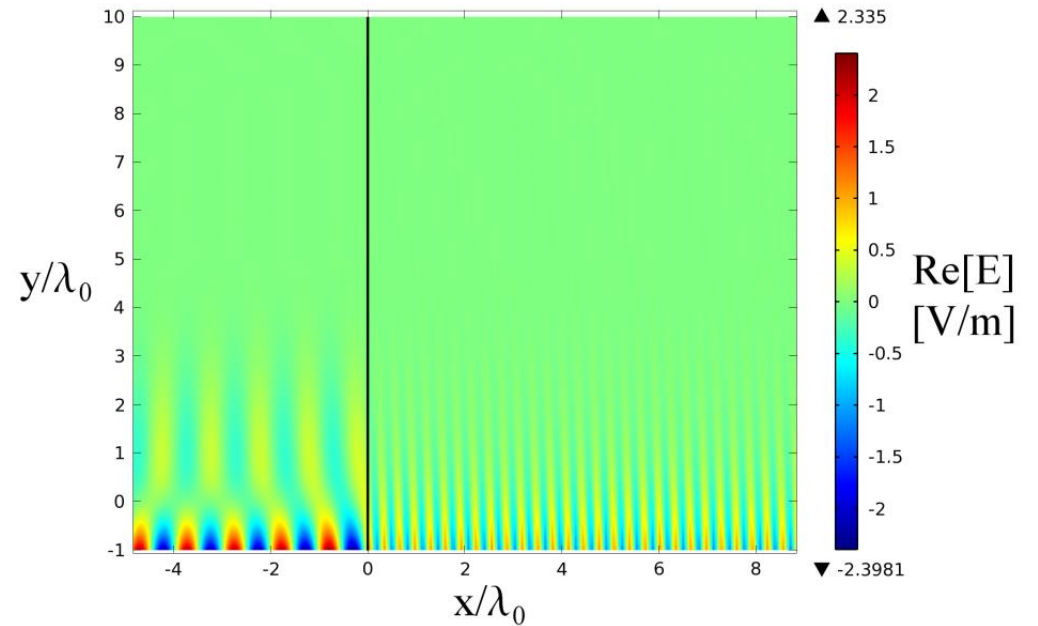
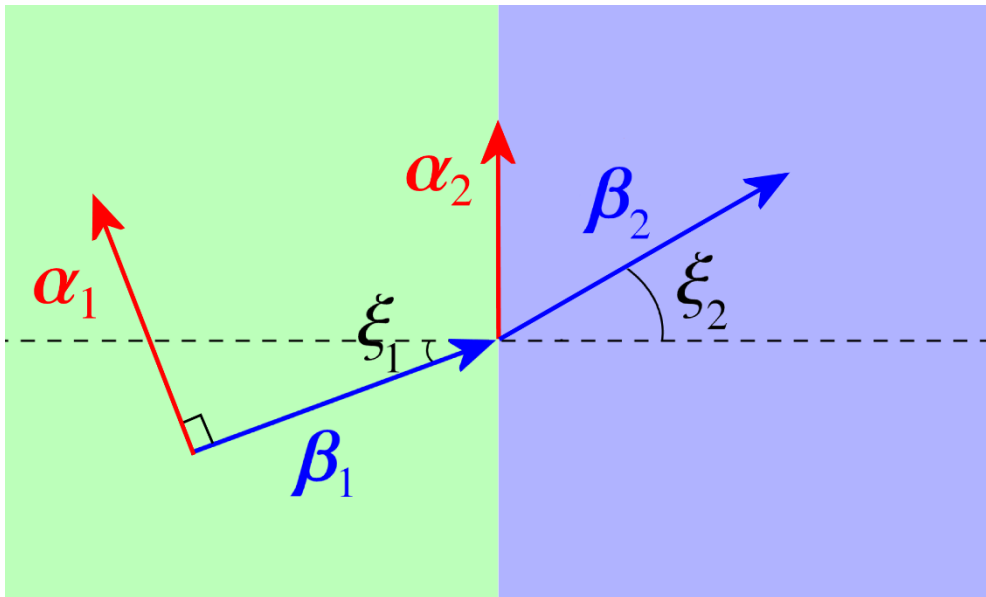
$$2\beta_2\alpha_2 \cos(\xi_2 - \xi_2) = k_0^2 \varepsilon_2'' \quad \Rightarrow \quad 2 \frac{\beta_1 \alpha_1 \sin \xi_i \cos \xi_i}{\sin \xi_t} \sin \xi_t = k_0^2 \varepsilon_2''$$

With some algebra:

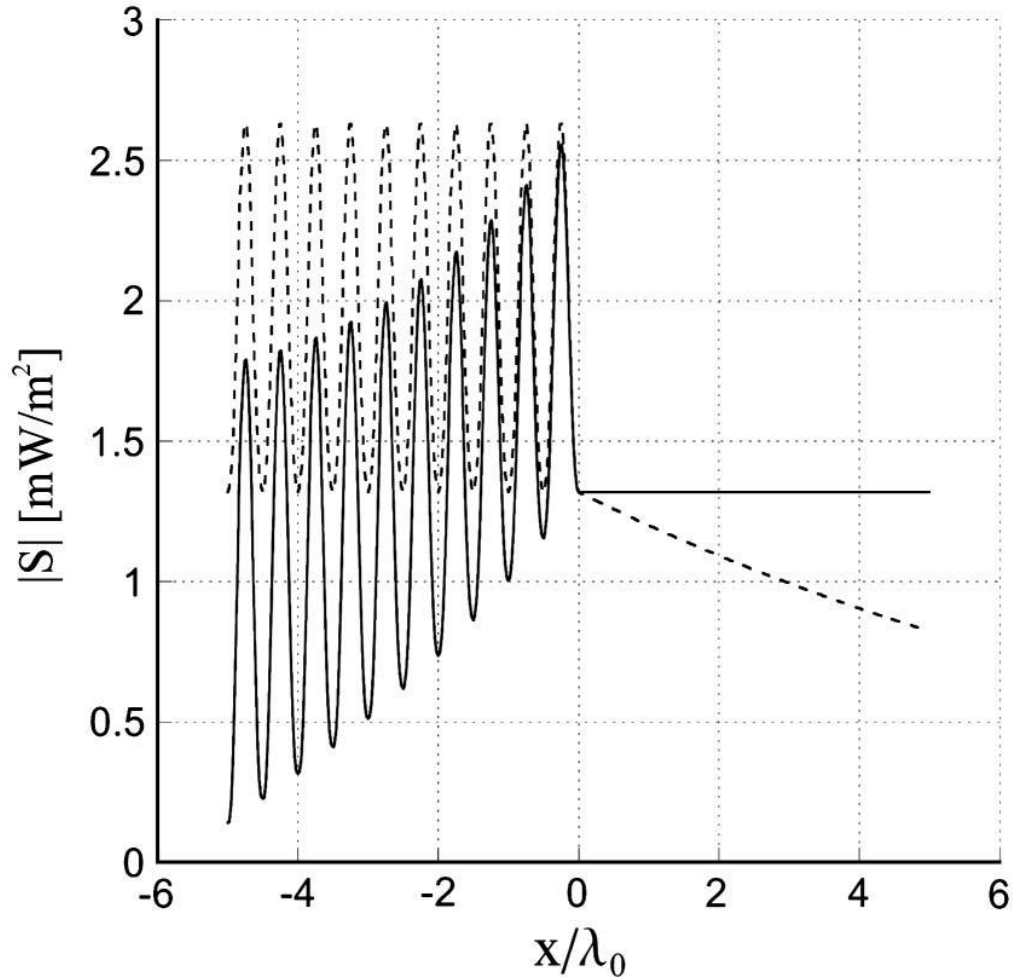
$$\beta_1 \alpha_1 \sin(2\xi_i) = k_0^2 \varepsilon_2'' \quad \Rightarrow \quad \xi_{DPW} = \frac{1}{2} \arcsin\left(\frac{k_0^2 \varepsilon_2''}{\beta_1 \alpha_1}\right)$$

Deep-Penetrating Wave (3)

At this point, we have an instrument to deeply penetrate in a dissipative medium coming from a lossless medium, as the air.



Deep-Penetrating Wave (4)



Comparing the magnitude of the Poynting vector of both the homogeneous and the inhomogeneous wave, we see:

- A sinusoidal oscillation in the first medium, because of the reflections;
- A decay of the transmitted power for the homogeneous incident wave;
- A constant transmitted power for the inhomogeneous wave

DPW requirements

To achieve the deep penetration, the incident inhomogeneous wave must have strong requirements:

- Its phase vector must have a minimum critical amplitude β_c , depending on the electromagnetic properties of the materials (k_1 and k_2 are the wavenumbers of the two materials):

$$\beta_1 \geq \beta_{1c} = \frac{k_1}{\sqrt{2}} \sqrt{1 + \sqrt{1 + \left[\frac{2\text{Im}(k_2^2)}{k_1^2} \right]^2}}$$

- The angle of the incident phase vector must have a well determined critical value:

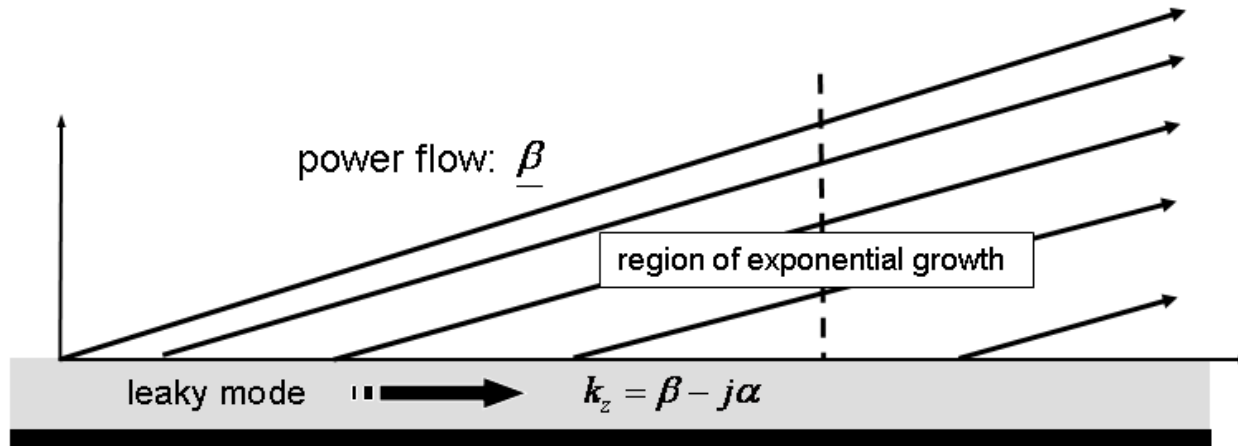
$$\xi_1 = \xi_{DPW} = \frac{1}{2} \arcsin \left(\frac{k_0^2 \varepsilon_2''}{\beta_1 \alpha_1} \right)$$

Antennas to achieve inhomogeneous waves

Good candidates antennas to achieve inhomogeneous waves are the Leaky-Wave Antennas (LWAs), able, in the near field, to generate inhomogeneous waves

The design of the LWA presents uncommon requirements:

usually the relevant design goals are the far-field properties, as the pointing angle or the beam width.

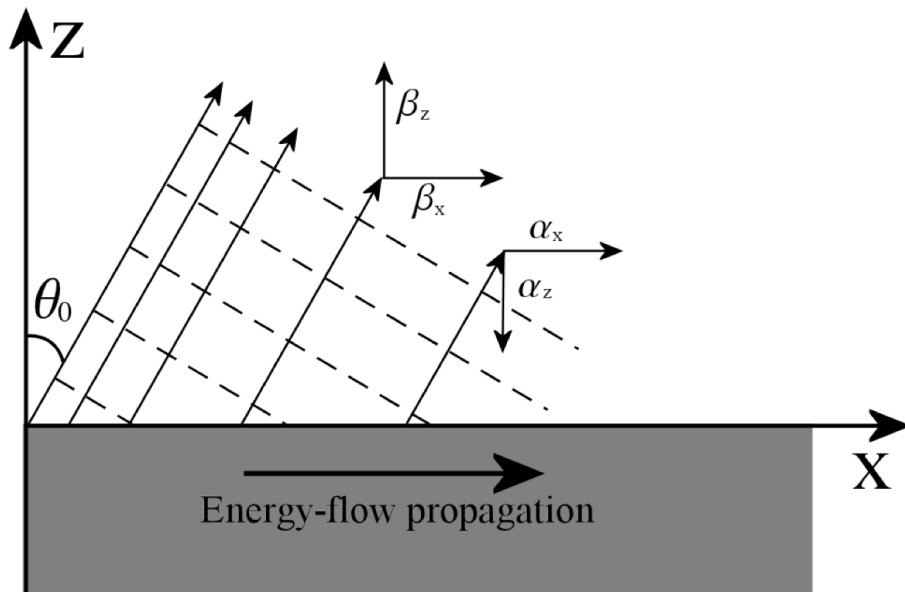
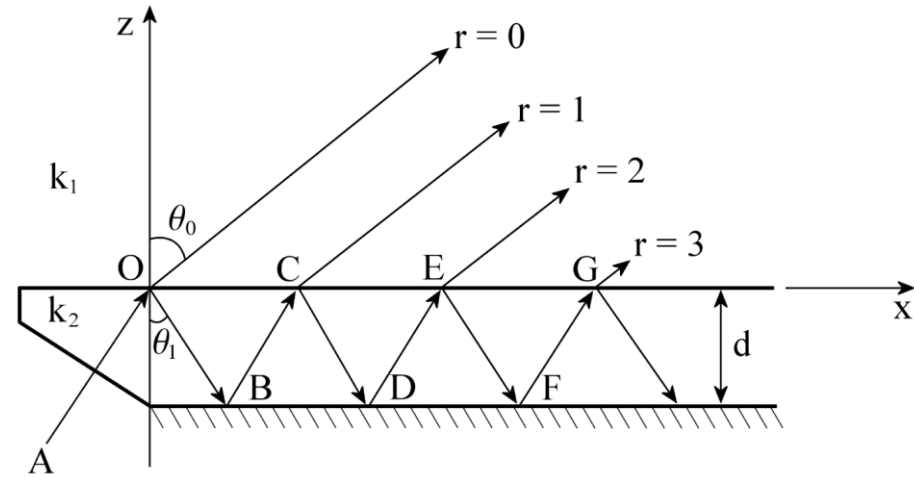


In our case, we are interested in the near-field properties:

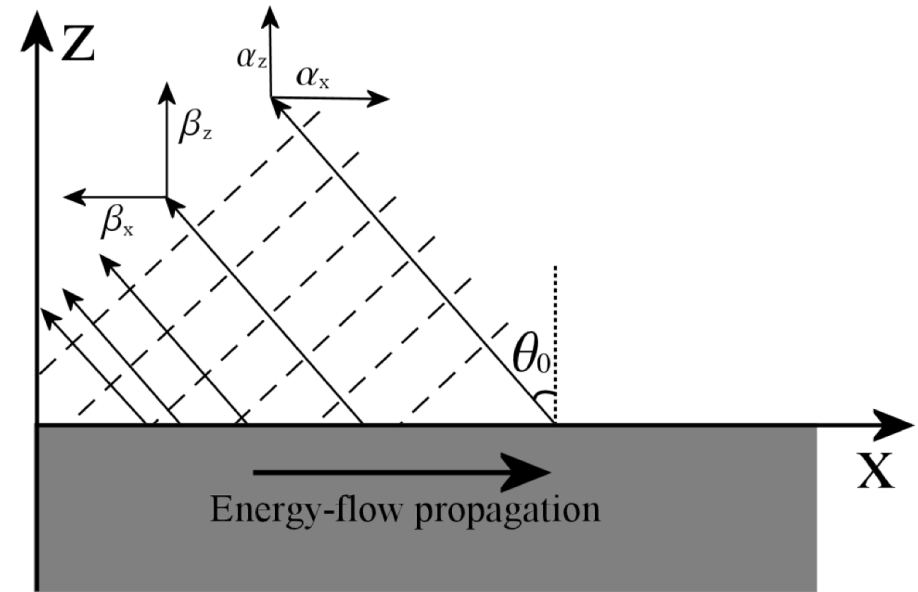
- Angle of the phase vector
- Magnitude of the generated phase vector.

Leaky-Wave Antennas

LWAs works by leakage from a guiding open structure:



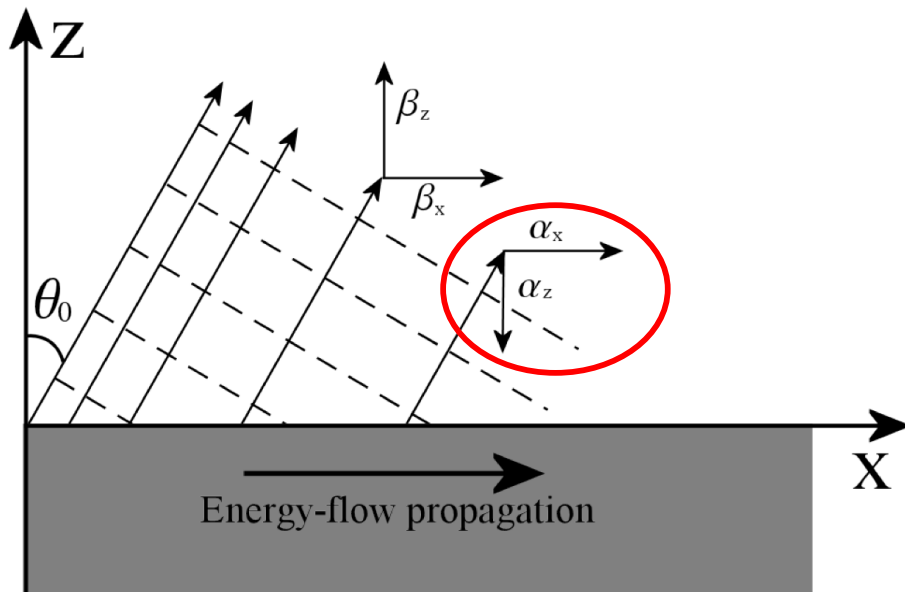
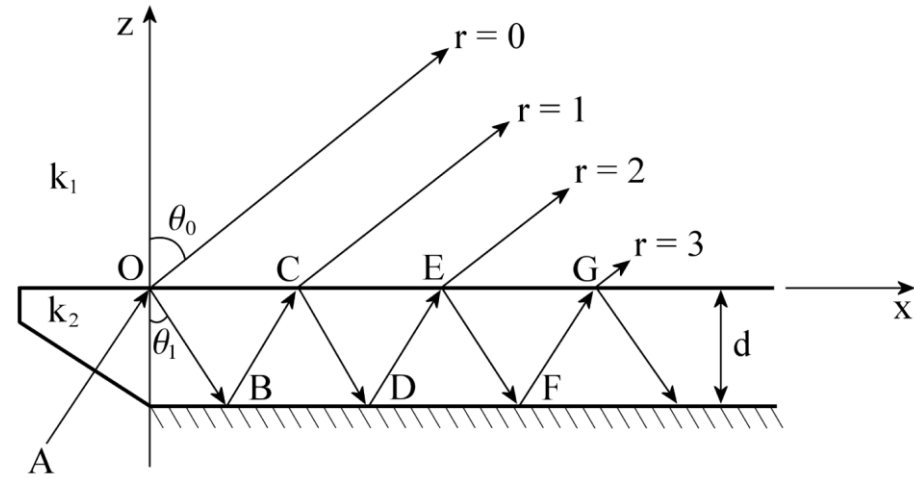
Forward Leaky wave: improper wave



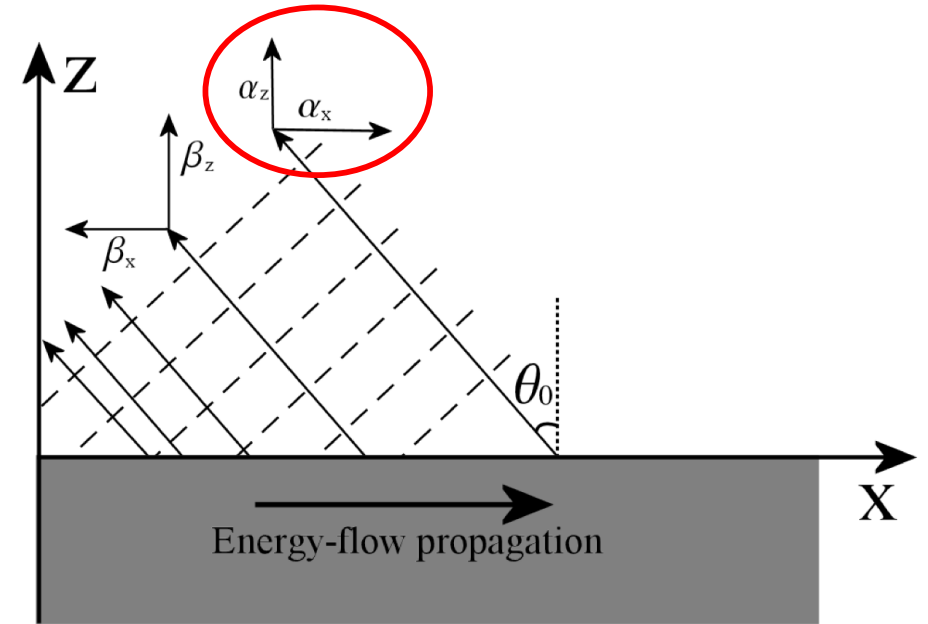
Backward Leaky wave: proper wave

Leaky-Wave Antennas

LWAs works by leakage from a guiding open structure:



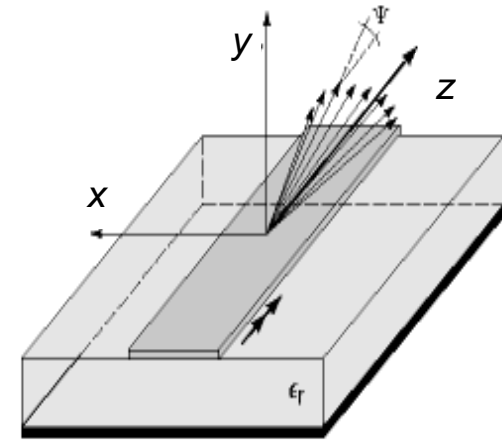
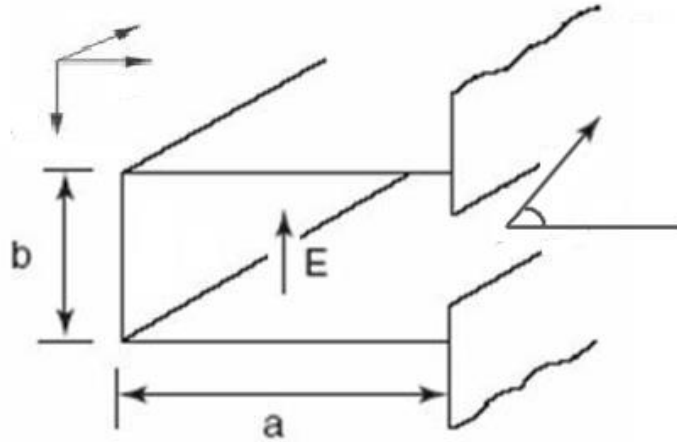
Forward Leaky wave: improper wave



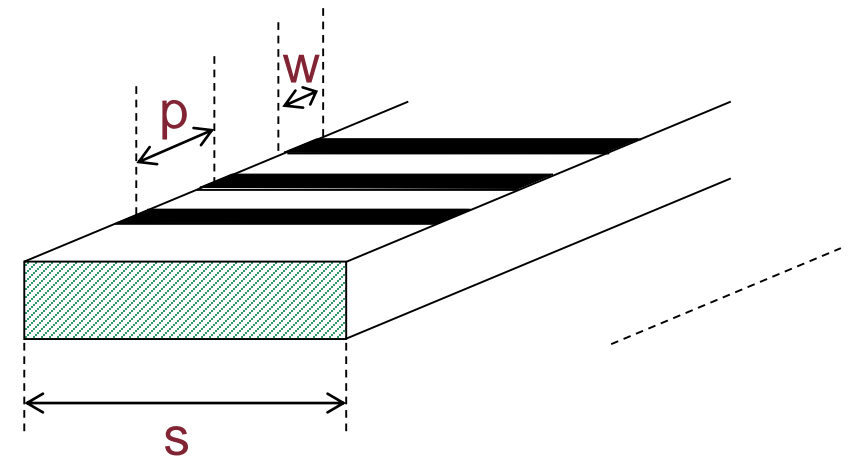
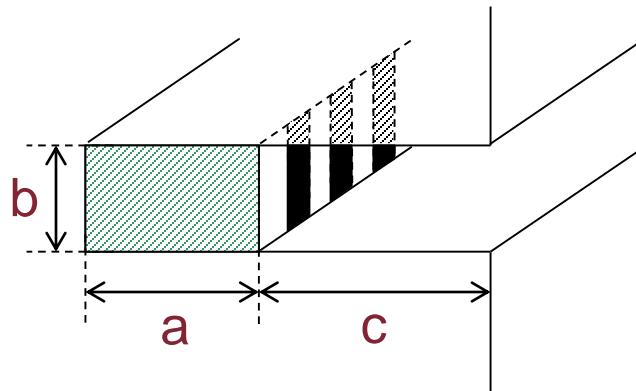
Backward Leaky wave: proper wave

Leaky-Wave Antennas (2)

Open waveguides:



Periodic structures:

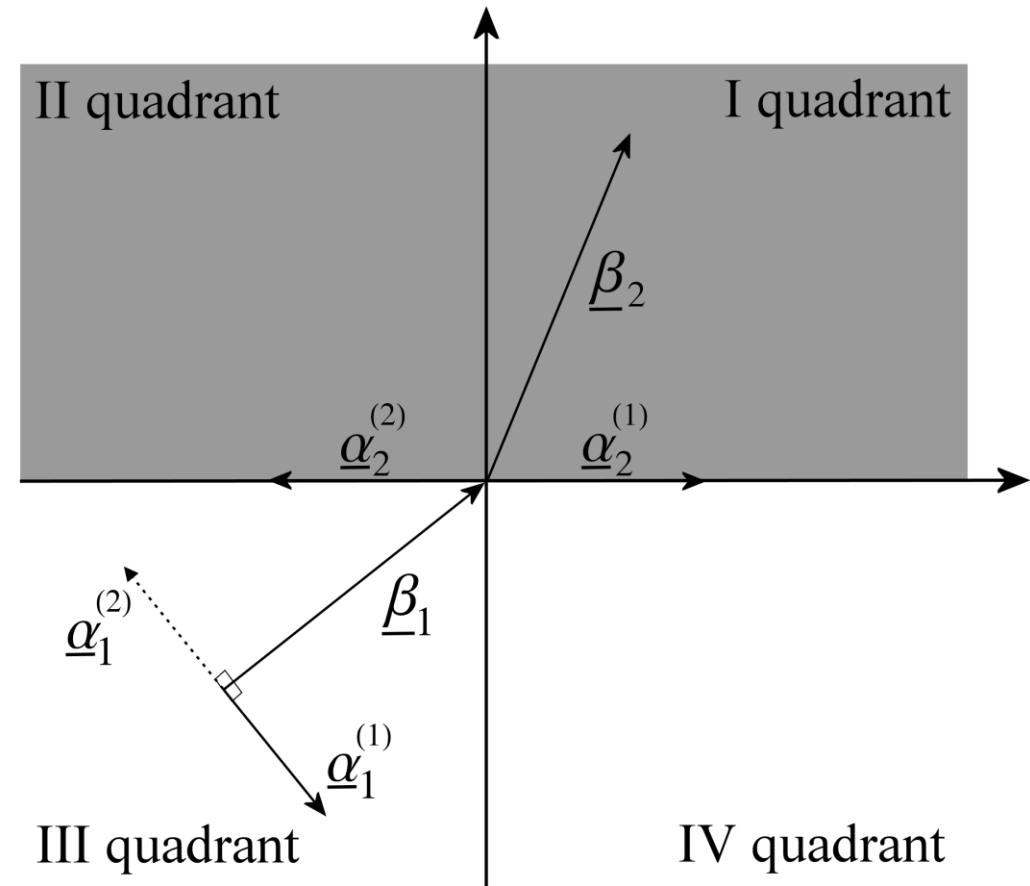


LWA and DPW

At this point we want to show how to obtain the DPW with a practical antenna.

To facilitate the measurement of the field is important to have the lossy medium separation surface parallel to the antenna aperture.

The Leaky Wave generated must be of the improper type!



Antenna design (1)

The medium 2 had to be chosen to reduce reflections which would have affected the amplitude of the electromagnetic field samples in the simulation.

The leaky-wave antenna has been designed to deeply penetrate at a given operating frequency in a well defined lossy medium:

$$f = 12 \text{ GHz} \quad \epsilon_{r2} = 1 \quad \mu_{r2} = 1 \quad \sigma_2 = 0.05 \text{ S/m}$$

Producing the following requirements for the incident inhomogeneous (leaky) wave:

$$\frac{\beta_1}{k_0} \approx 1.003 \quad \frac{\alpha_1}{k_0} \approx 0.075 \quad \xi_c = \frac{\pi}{4}$$

In terms of longitudinal normalised phase and attenuation constants of the leaky-wave antenna:

$$\frac{\beta_z}{k_0} \approx 0.71 \quad \frac{\alpha_z}{k_0} \approx 0.053$$

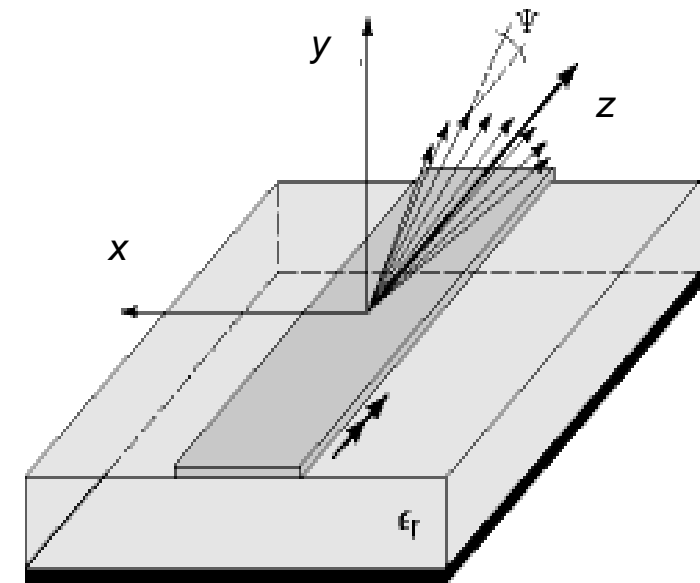
Antenna design (2)

A simple, uniform and planar leaky-wave antenna (LWA) was selected, and in particular a **microstrip LWA**, to prove the deep-penetration in the chosen medium.

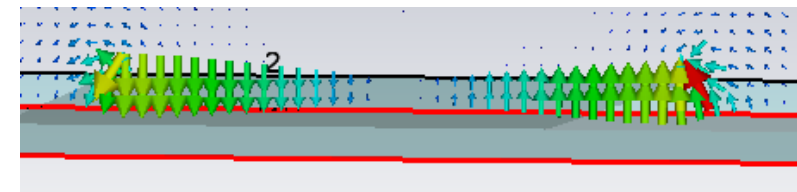
The microstrip LWA has been designed to radiate in the first-order leaky mode (EH_1), with longitudinal normalised phase and attenuation constants

$$\frac{\beta_z}{k_0} = 0.7088 \quad \frac{\alpha_z}{k_0} = 0.05234$$

Very close to the required ones



Antenna Layout



Port mode (Odd)

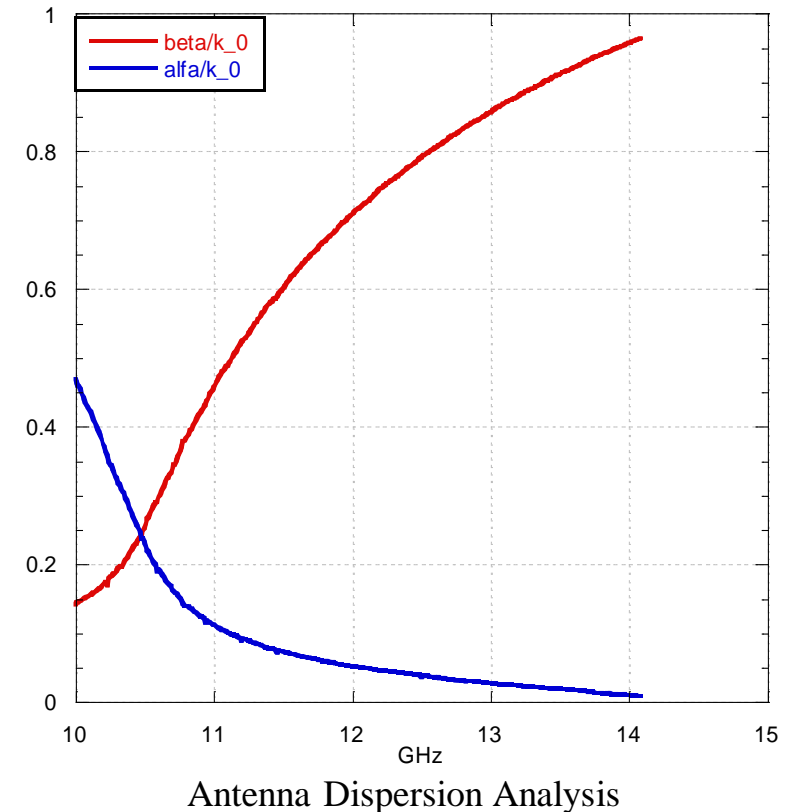
Antenna design (3)

A Method-of-Moments numerical method for the modal dispersion analysis of uniform microstrip structures has been used for the antenna design.

The antenna length has been chosen equal to $L = 5\lambda = 0.125$ m, in order to radiate the 96% of the injected power, approximately.

To obtain the length value, the following formula has been applied:

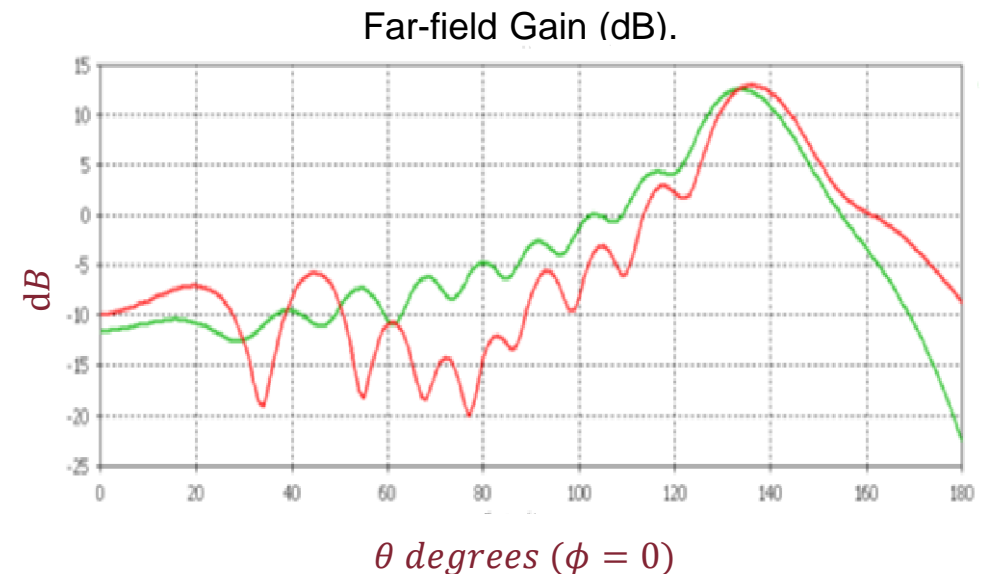
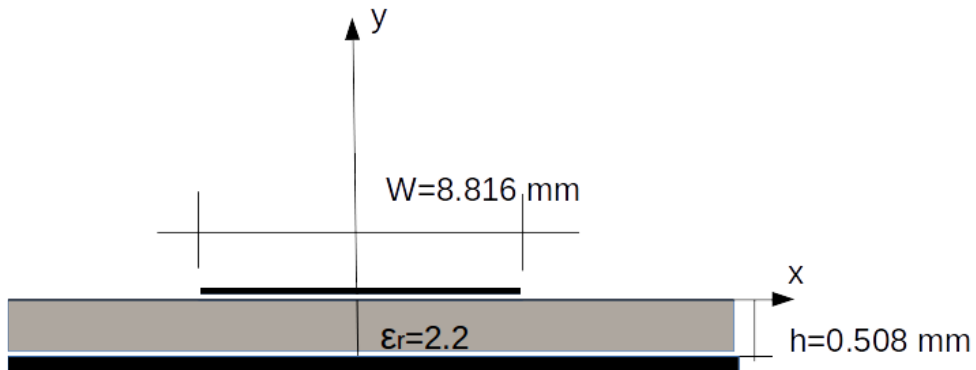
$$L = -\frac{\lambda_0}{4\pi \frac{\alpha_z}{k}} \ln \left[\frac{P(L)}{P(0)} \right] \approx 0.122 \text{ m}$$



Antenna design (4)

The antenna has been simulated on a commercial software (CST Microwave Studio) to verify compliance of the requirements.

The simulations have been performed in both the frequency and the time domains.



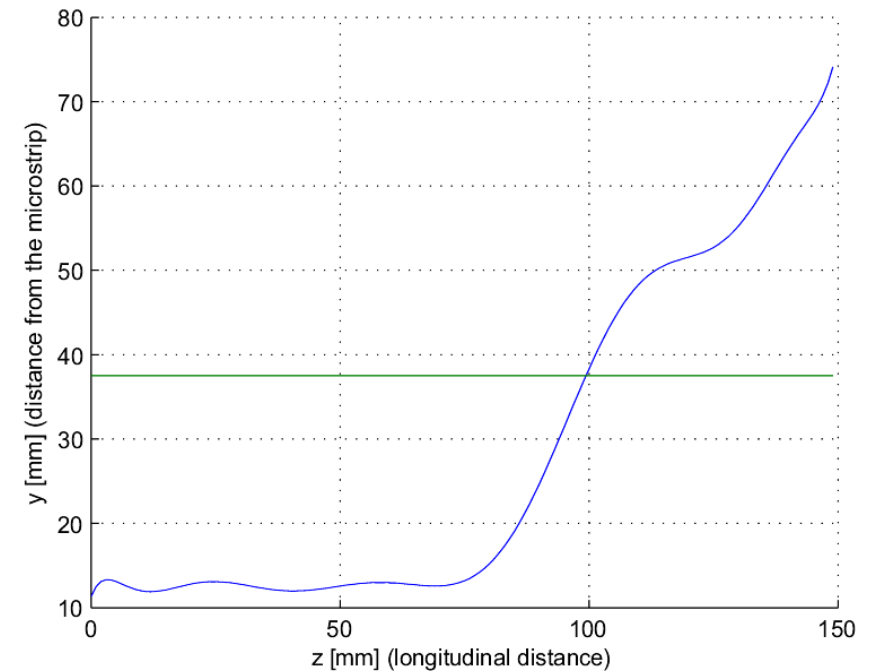
Magnitude of the far-field gain obtained by electromagnetic simulations using both frequency domain (green curve) and time domain (red curve) methods.

Simulation domain

In order to verify the penetration depth in the lossy medium, the space gap between the antenna and the interface has been accurately chosen.

The interface is located at $\frac{3}{2}\lambda$ above the microstrip antenna.

The lossy-medium interface intercepts the curve describing the maxima of the electric field in proximity of $z = 4\lambda$



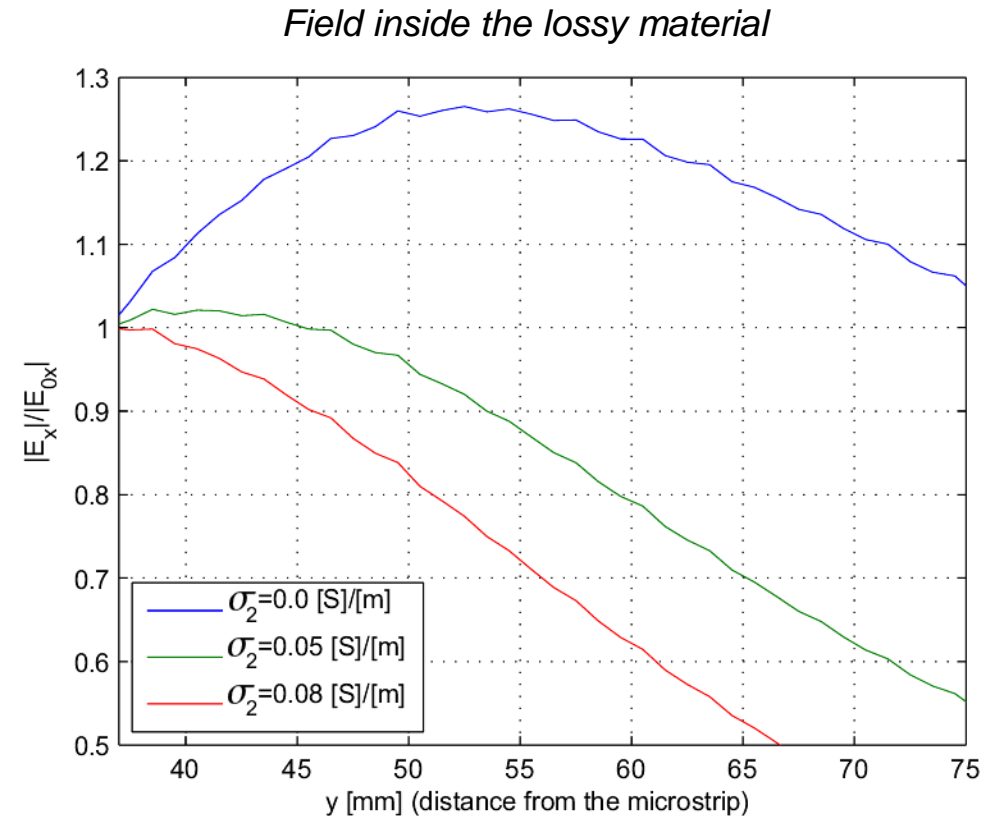
Magnitude of the maximum value of the electric field $|E_x|$ (blue line) and position of the planar interface (green line) .

Deep-penetration evaluation

The magnitude of the electric field, normalized with respect to its value on the interface, inside the lossy material has been evaluated as a function of the depth.

Simulations have been implemented in three different cases:

- a lossless material, $\sigma_2 = 0 \text{ S/m}$
- the lossy material used for the design, $\sigma_2 = 0.05 \frac{\text{S}}{\text{m}}$
- a lossy material with higher conductivity, $\sigma_2 = 0.08 \text{ S/m}$

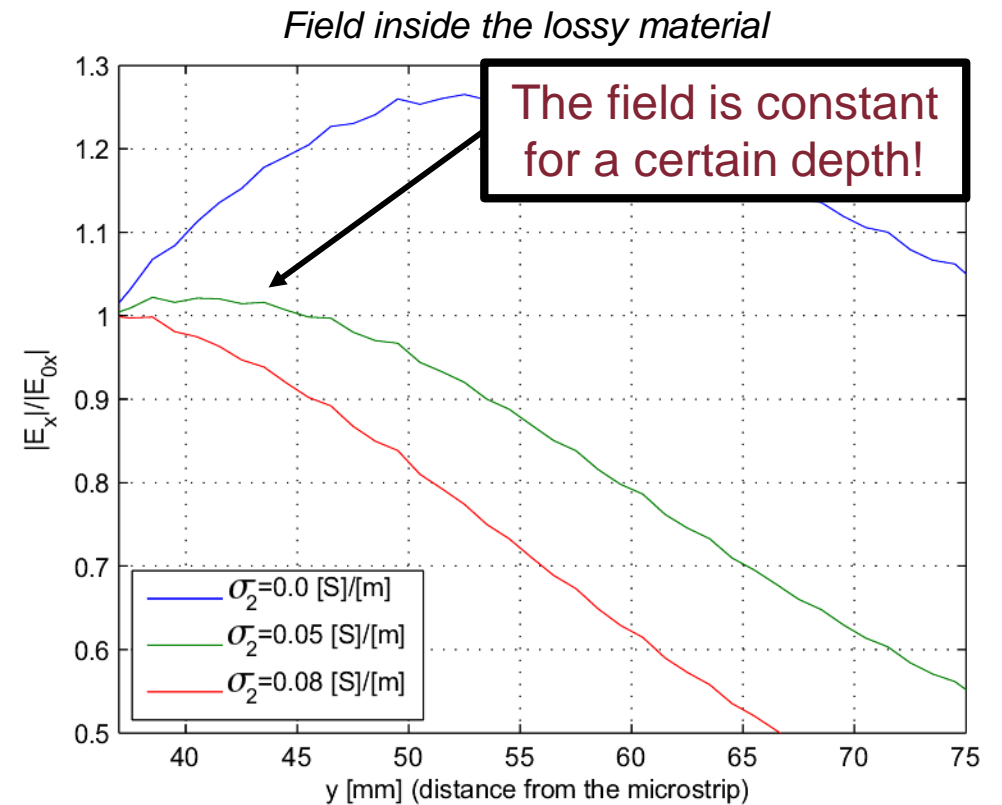


Deep-penetration evaluation

The magnitude of the electric field, normalized with respect to its value on the interface, inside the lossy material has been evaluated as a function of the depth.

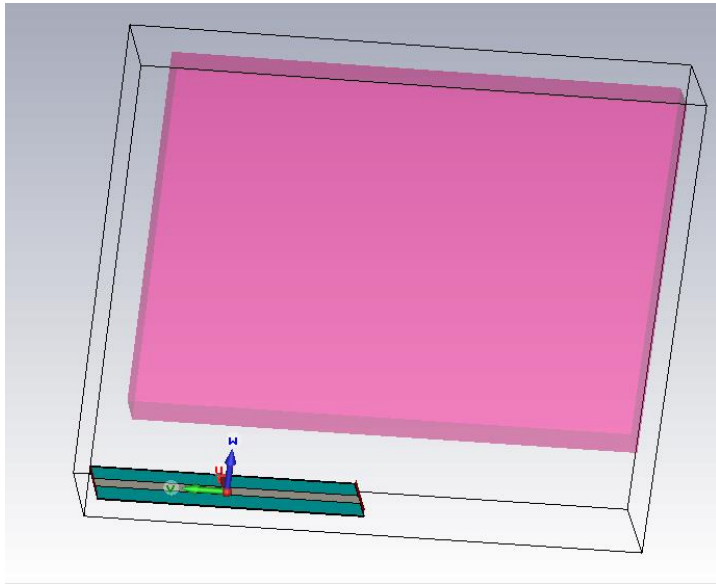
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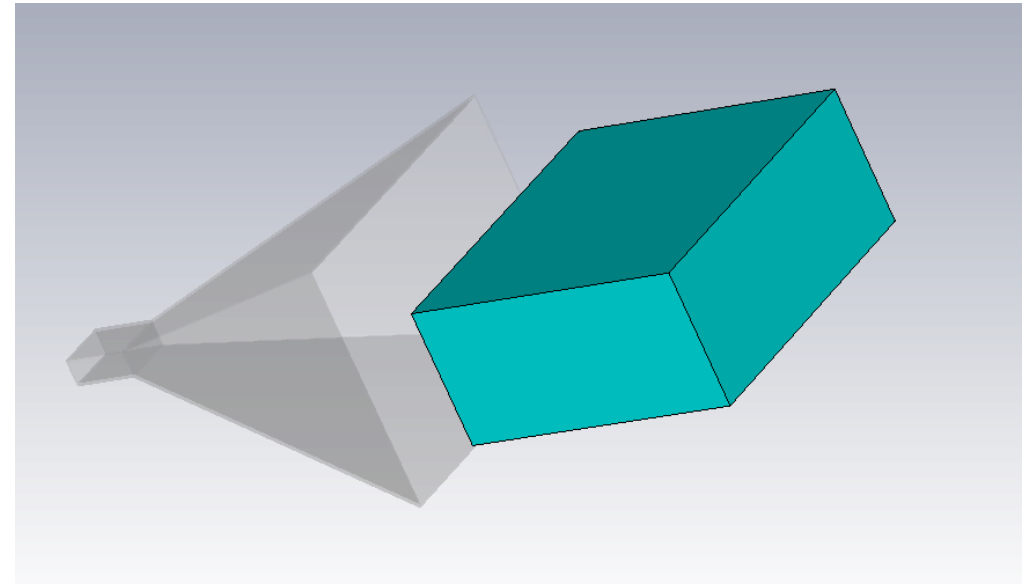


Deep-penetration evaluation (2)

The result presented can be compared with the penetration obtained with a conventional horn antenna



LWA and lossy medium

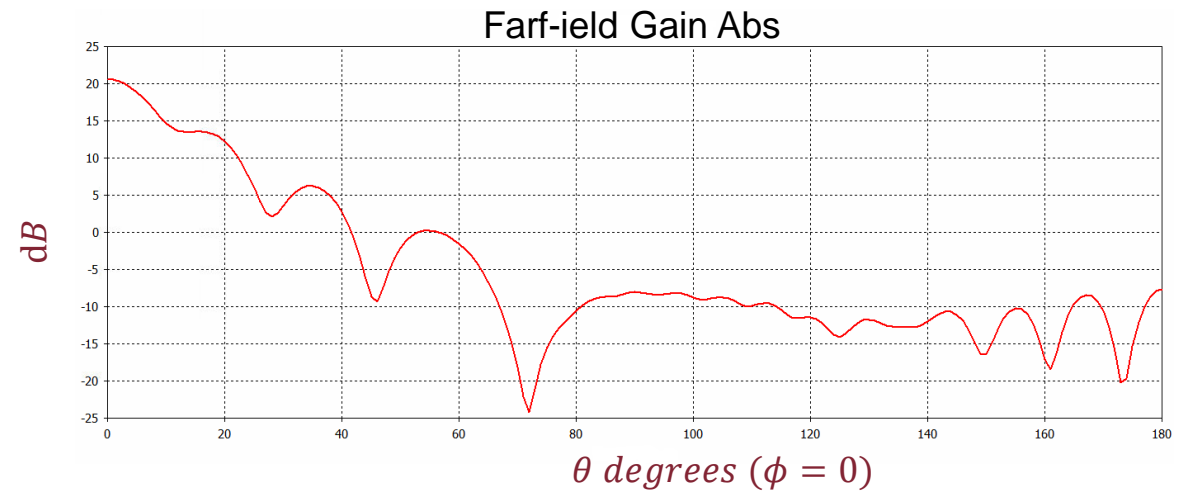
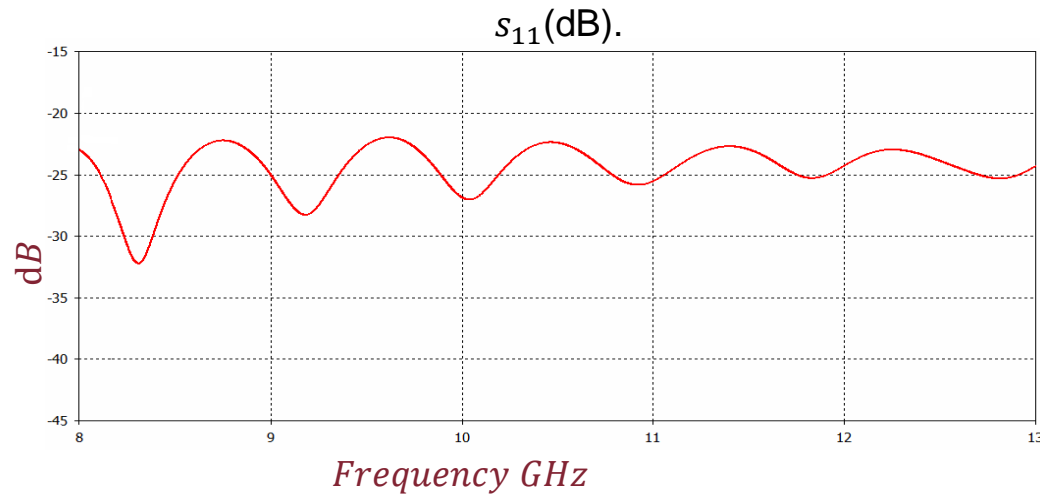


Horn Antenna with lossy medium

In both cases the lossy medium is placed at $1,5\lambda$ from the aperture and its base is parallel to the antenna aperture.

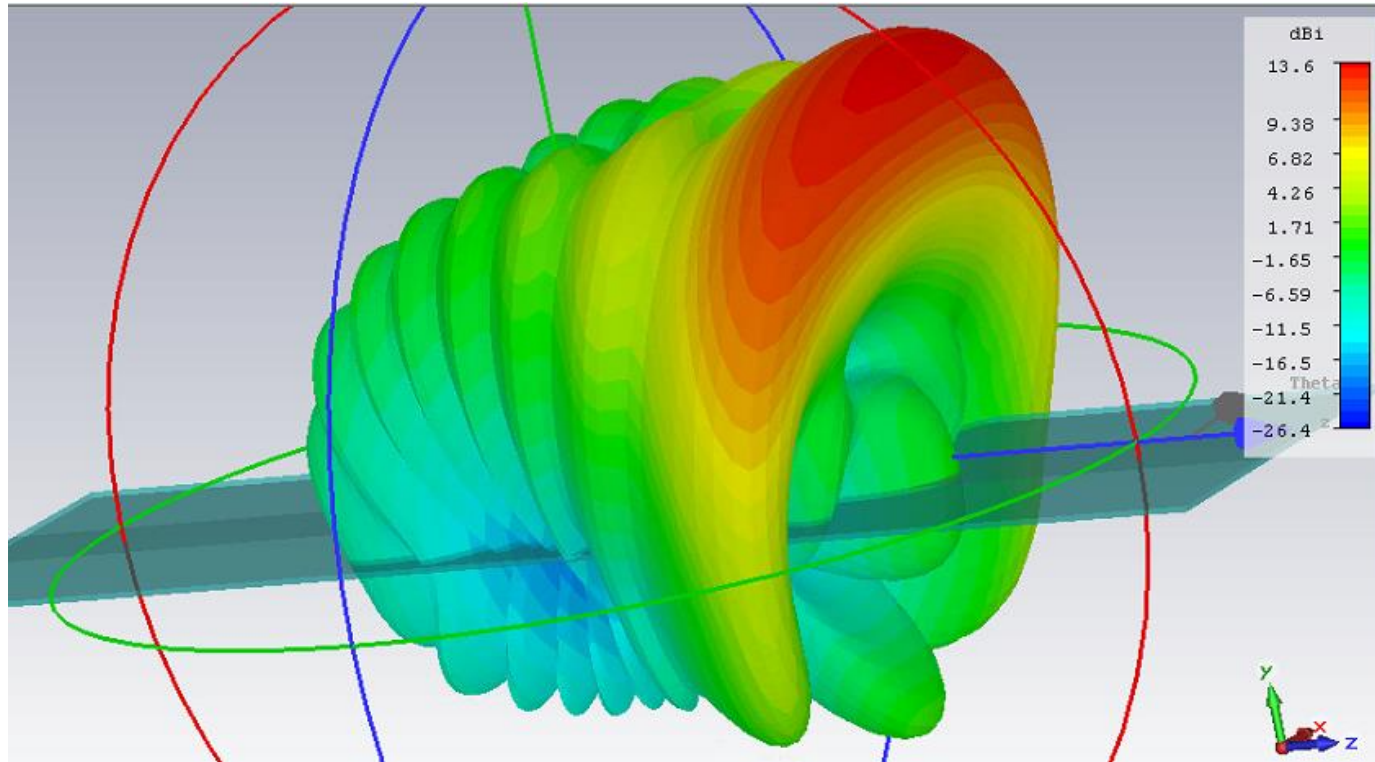
Deep-penetration evaluation (3)

Horn Antenna parameters.

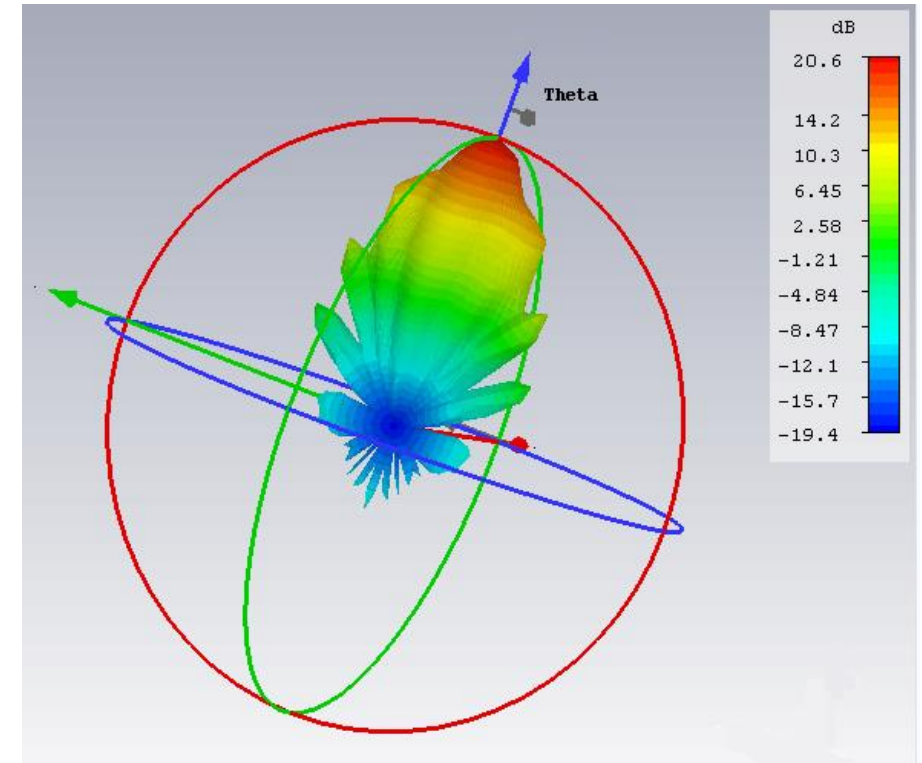


The Horn Antenna designed for the comparison presents high gain and it is broadband.

Deep-penetration evaluation (4)



LWA 3D Far-field

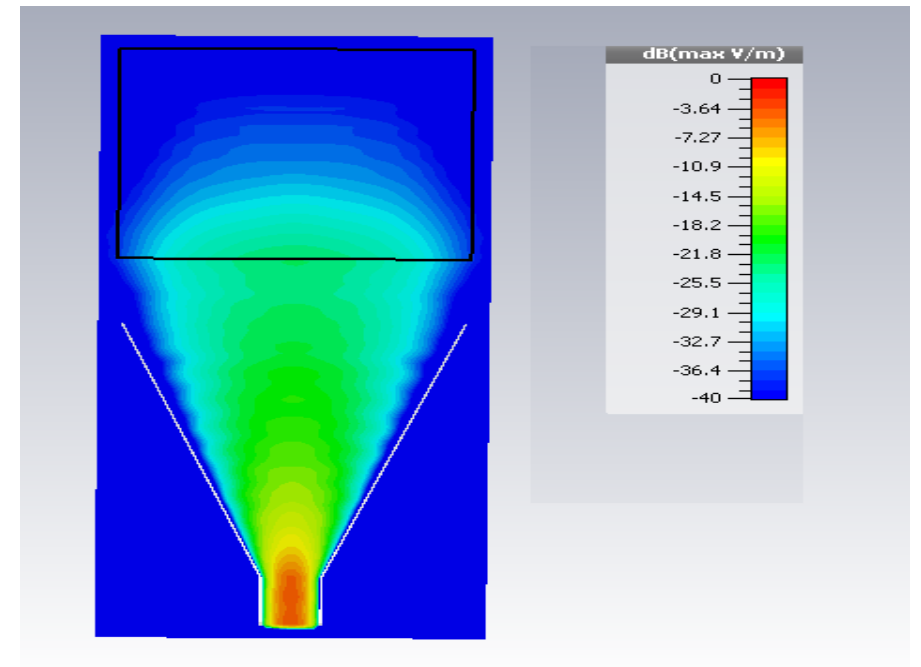
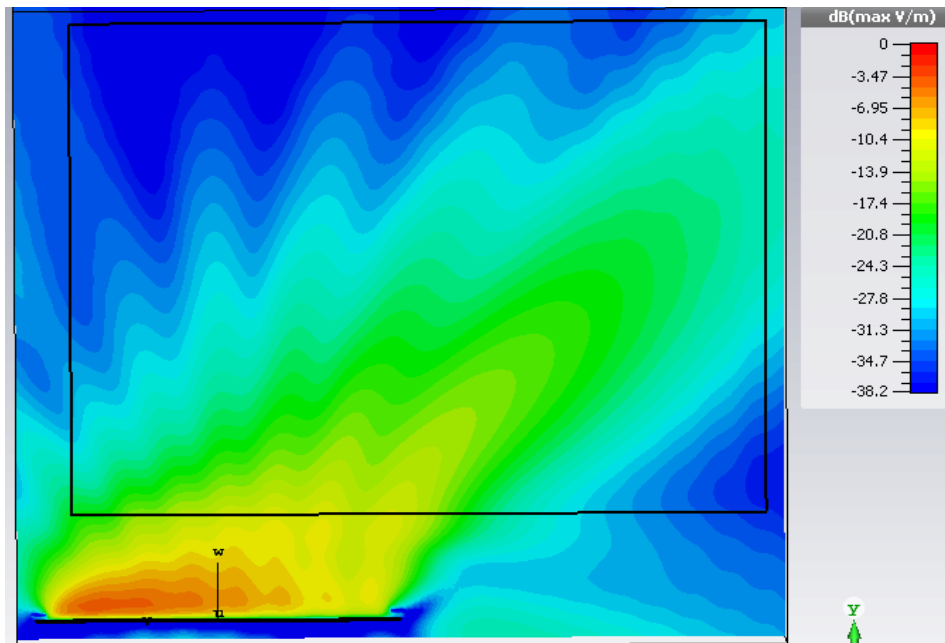


Horn antenna 3D Far-field

3D Far Field comparison between the two antennas.

Deep-penetration evaluation (5)

The result presented for the LWA can be compared with the penetration obtained with a conventional horn antenna



Mean-Magnitude of the electric field $|E_x|$ for a microstrip LWA (left) and a rectangular horn antenna (right)
computed in dB from the generated E-Field

Averaged fields

To allow a fair comparison between different antenna types (and between different LWA longitudinal sections) an algorithm which would take in consideration mediated fields had to be developed for the LWA.

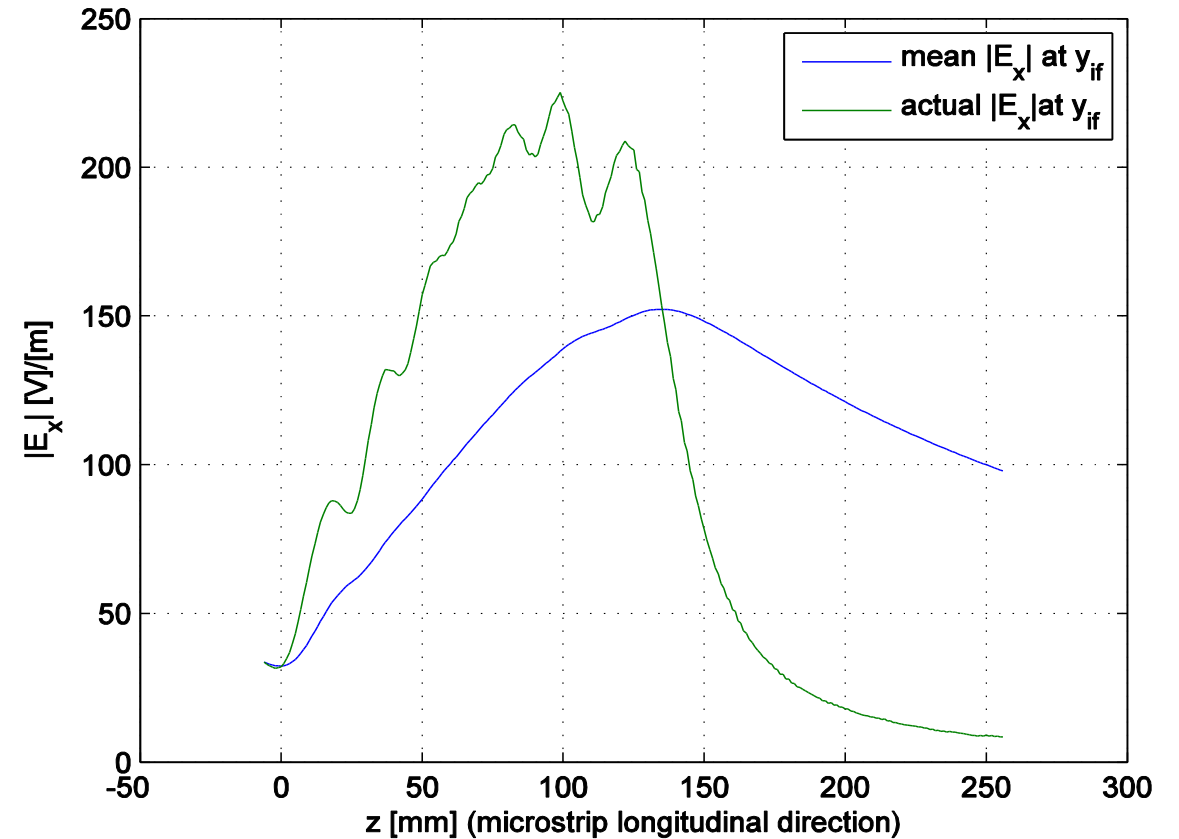
The electric field produced by the LWA antenna has been mediated on the longitudinal direction z on k samples:

$$\overline{E}^{LWA}(0, y_i, z_k) = \frac{E^{LWA}(0, y_i, z_k)}{\sum_{j=0}^{j=k-1} E_{y_{if}}^{LWA}(0, y_{if}, z_j)/k} \quad \forall y_i > y_{if}.$$

The amplitude of the electric field in a point of the lossy medium at a given z_k is normalized by the mean field calculated considering all the samples of the Electric field amplitude on the separation surface that precede the considered point (i.e. for $j < k$).

Averaged fields (2)

Here we can see the difference between the actual magnitude of the electric field and the average magnitude for a given penetration depth



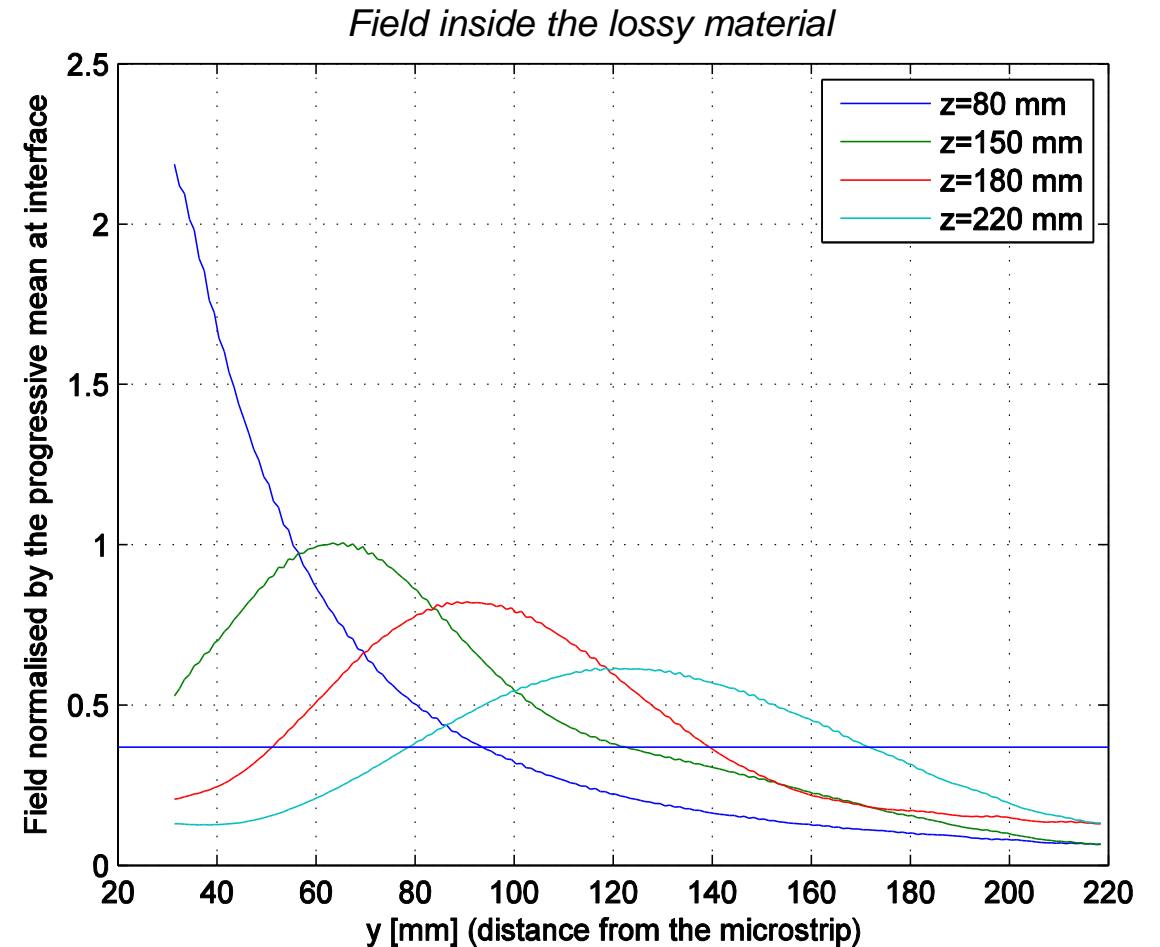
Deep-penetration comparison

Normalized electric field amplitude as a function of the penetration depth for different longitudinal positions

Horizontal line represents the value $1/e$

Electric field tends to penetrate more for higher z values

For $z = 80$ mm the normalized electric field stays above $1/e$ only for 45 mm, but it stays above such a value for about 80 mm at $z = 220$ mm



Averaged global fields (1)

A global mean was also developed. This permitted a penetration “global” mediated comparison between the two fields.

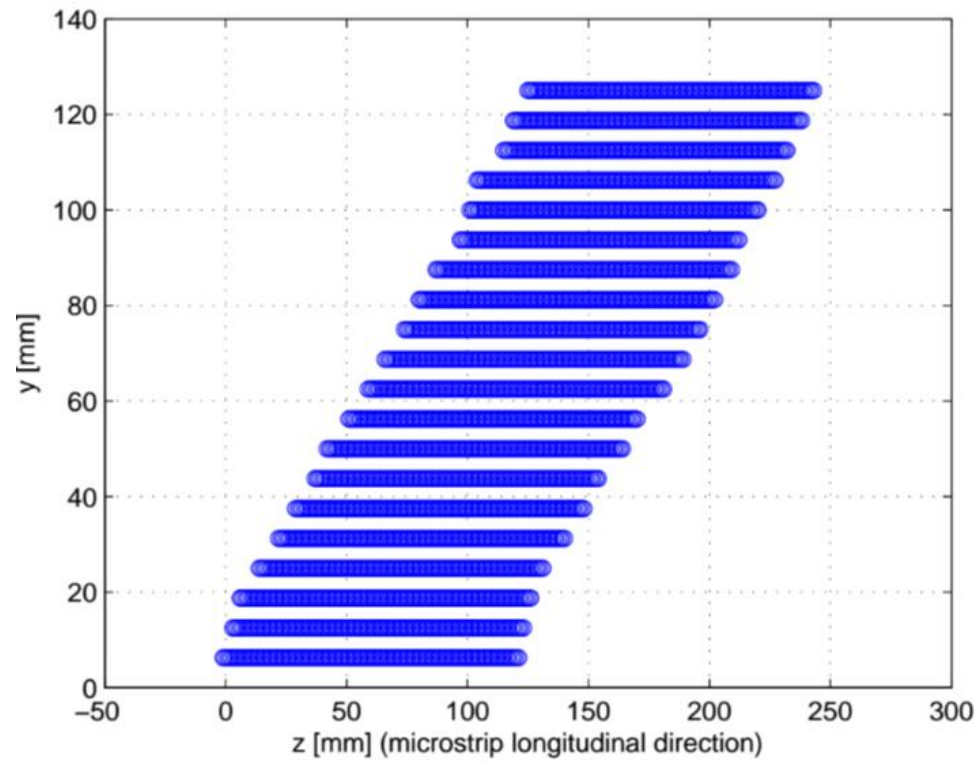
The electric field produced by the antennas has been mediated on the longitudinal direction z on N samples taken at a distance of 1 mm, N is estimated as the number of samples at the aperture for which the electric field is reduced by a factor 0.16 of its maximum.

$$\bar{E}(0, y_i) = \frac{\sum_{k=n}^{k=n+N} E(0, y_i, z_k)}{\sum_{j=0}^{j=N} E_{y_i}(0, y_{if}, z_j)} \quad \forall y_i > y_{if}; \quad y_i = y_{i-1} + 1\text{mm}$$

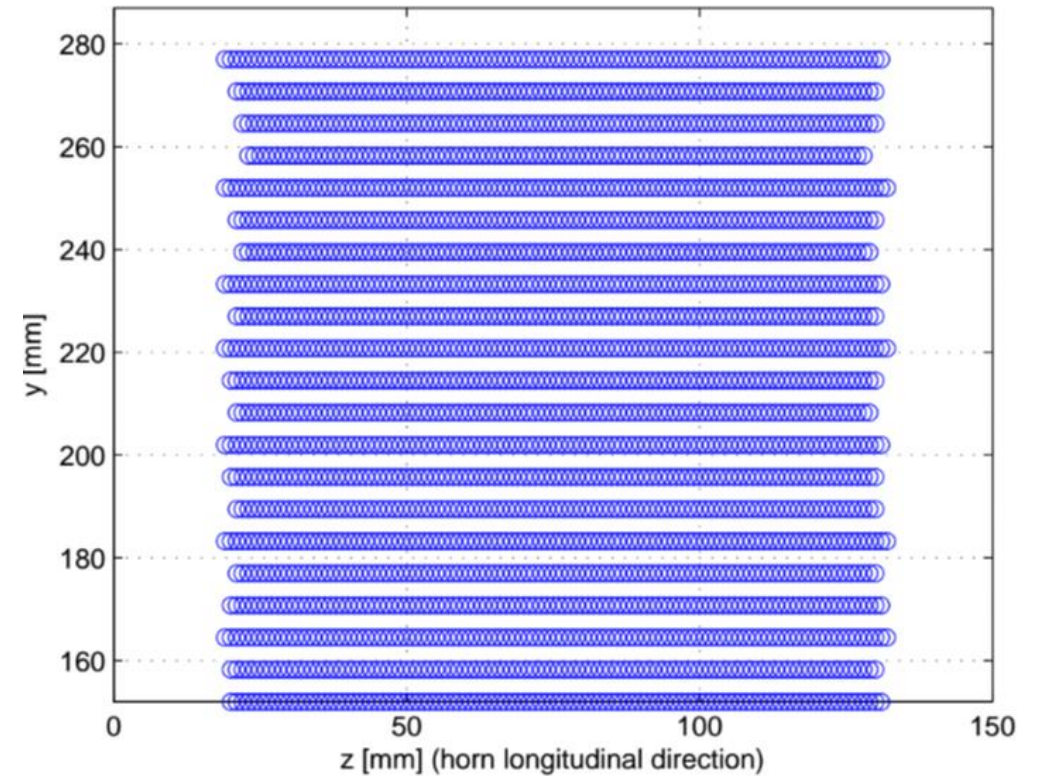
Electric field in a point of the lossy medium is normalized by the mean field calculated considering all the samples of the Electric field amplitude on the separation surface for which the field is maximum.

Averaged global fields (2)

Samples considered for both LWA and Horn



LWA case



Horn case

Deep-penetration comparison (2)

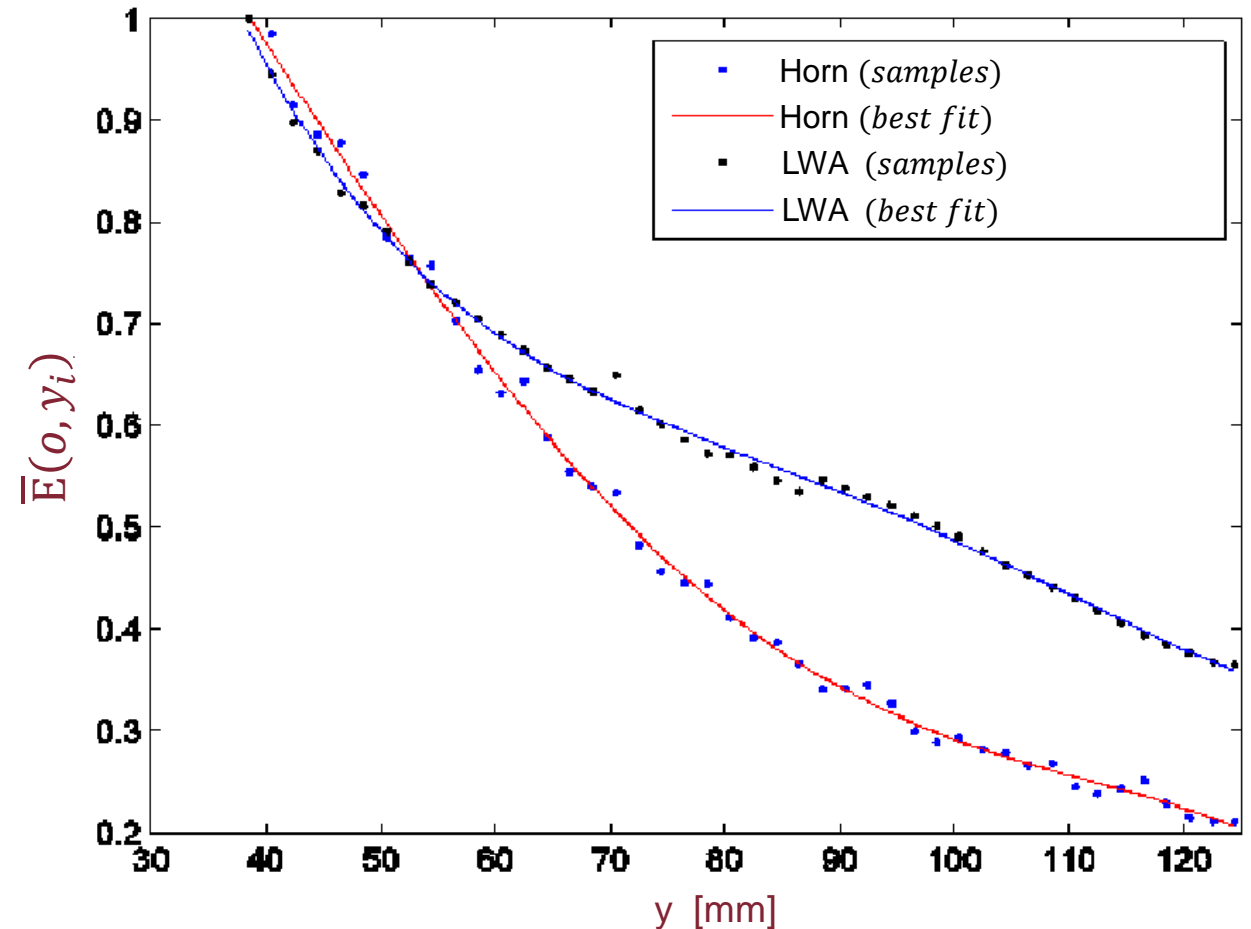
Here we analyze the penetration by considering the mediated electric field along z

The electric field is mediated along z on N samples

An algorithm searches the maximum of the electric field along z , for each y . All consecutive samples with maximum amplitude around this value are selected until the sum is equal to N

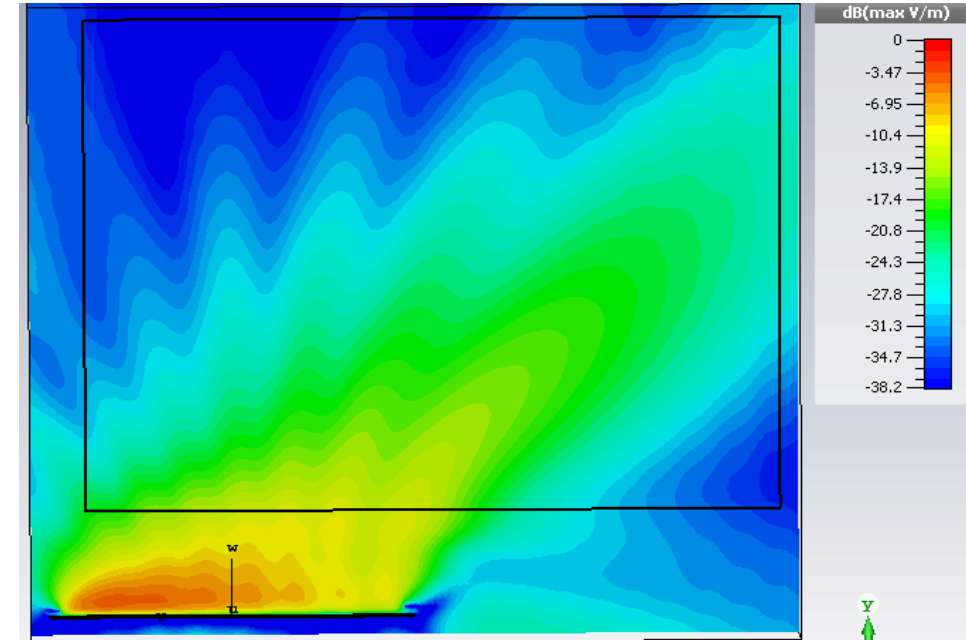
The field generated by the LWA is stronger than the one generated by the horn for $y > 80 \text{ mm}$

Comparison between averaged Horn and LWA penetration in a lossy medium ($\sigma = 0.05 \text{ [S]/[m]}$)



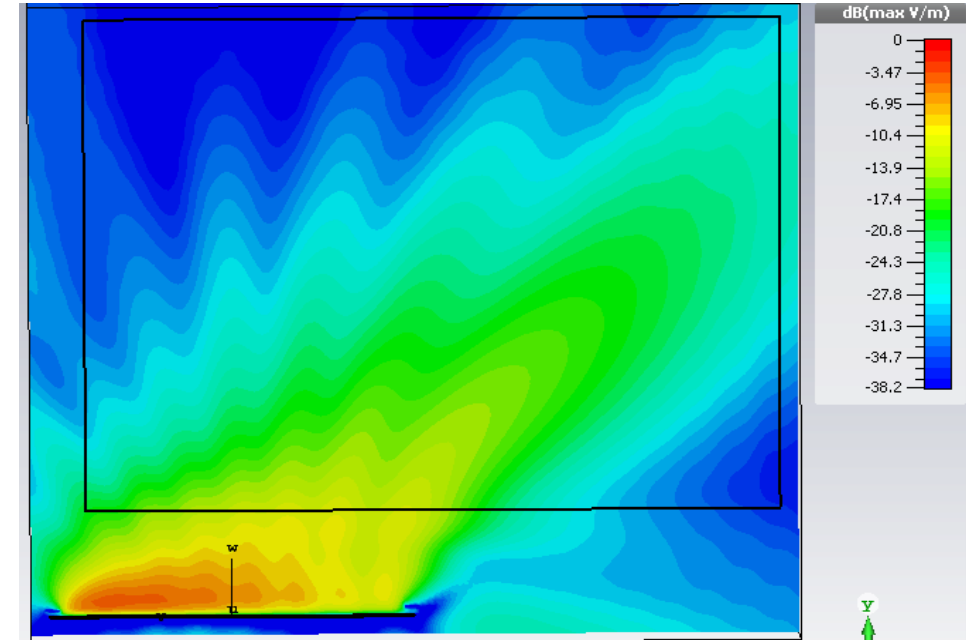
DPW with LWA: conclusions

- As we seen, a LWA allows to obtain the DPW in a lossy material
- The antenna design can be done with conventional methods
- Comparisons with conventional antennas show a greater amount of power transmitted in the lossy material
- A strong limitation comes from the value of the phase amplitude



DPW with LWA: conclusions

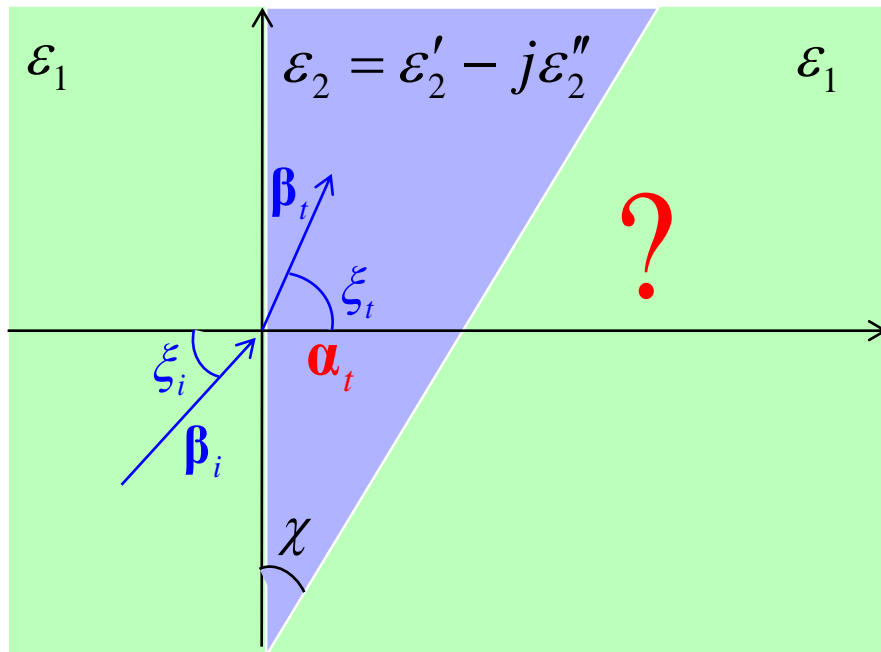
- As we seen, a LWA allows to obtain the DPW in a lossy material
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- A strong limitation comes from the value of the phase amplitude



We can wonder if it is possible to generate inhomogeneous waves with alternative mechanisms

Example

A lossy (two-dimensional) prism!



We can solve for **the first interface**:

$$\beta_2 = \sqrt{\frac{|\beta_{iy}|^2 + k_0^2 \varepsilon'_2 + |\beta_{iy}^2 - k_2^2|}{2}}$$

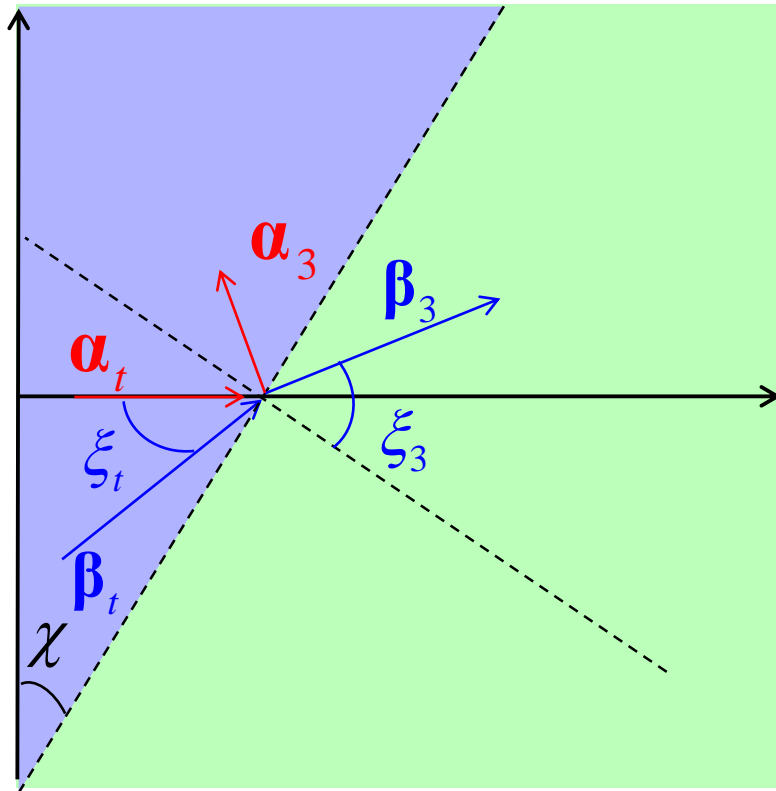
$$\alpha_2 = \sqrt{\frac{|\beta_{iy}|^2 - k_0^2 \varepsilon'_2 + |\beta_{iy}^2 - k_2^2|}{2}}$$

$$\xi_t = \arcsin\left(\frac{\beta_1}{\beta_2} \sin \xi_1\right)$$

Example (2)

The second interface, a new reference frame:

$$\begin{cases} \xi'_t = \xi_t + \chi \\ \zeta'_t = \chi \end{cases}$$

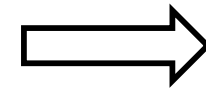


$$\beta_3 = \sqrt{\frac{|\beta'_{ty}|^2 + k_0^2 \varepsilon_1 + |\beta'_{ty}|^2 - k_1^2}{2}}$$

$$\alpha_3 = \sqrt{\frac{|\beta'_{ty}|^2 - k_0^2 \varepsilon_1 + |\beta'_{ty}|^2 - k_1^2}{2}}$$

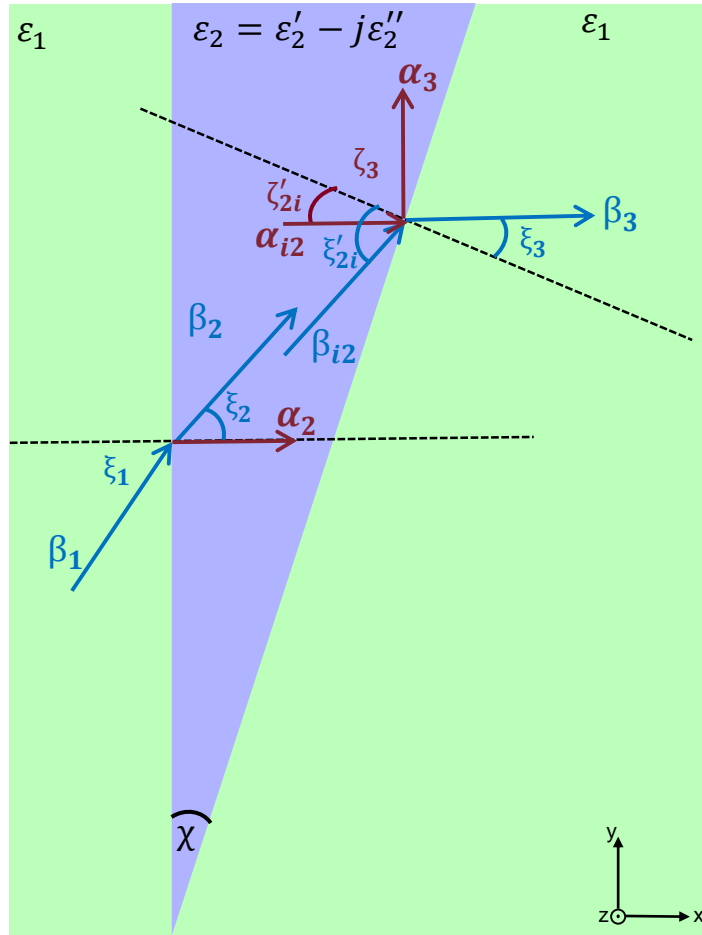
$$\xi_3 = \arcsin\left(\frac{\beta_2}{\beta_3} \sin \xi'_t\right)$$

$$\zeta_3 = \arcsin\left(\frac{\alpha_2}{\alpha_3} \sin \zeta'_t\right)$$



$$\zeta_3 - \xi_3 = \pm \frac{\pi}{2}$$

Example (3)



If a homogeneous wave from a lossless medium impinges on a dissipative prism (with two non parallel interfaces), the transmitted wave through the prism is an inhomogeneous wave in a lossless medium

This can be an alternative approach to obtain inhomogeneous waves for reach the DPW.

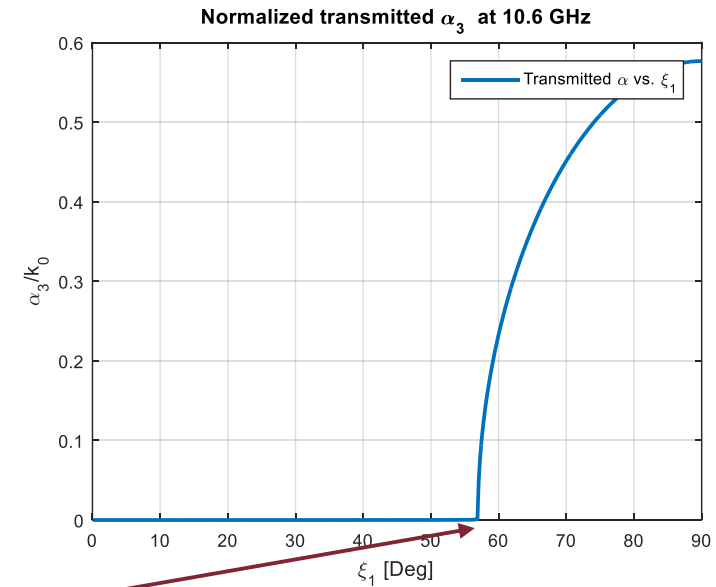
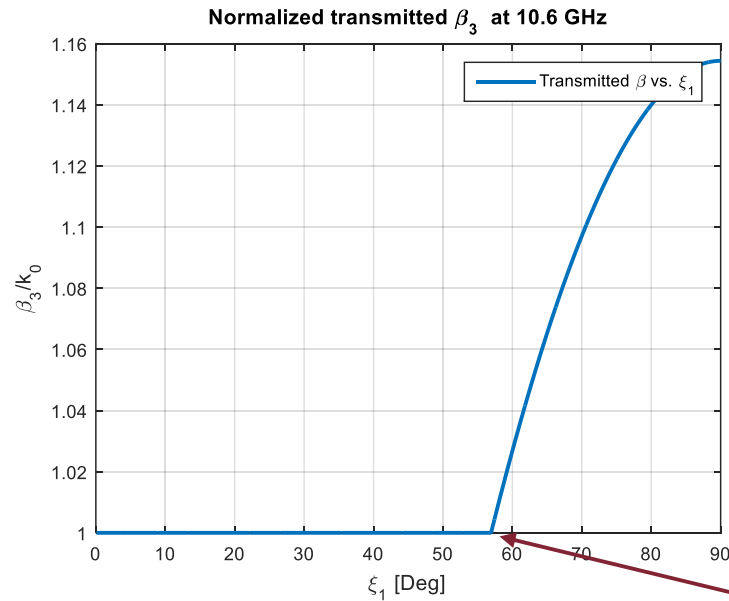
Transmitted wave characterization

Consider the following scenario:

$$f_0 = 10.6 \text{ GHz}$$

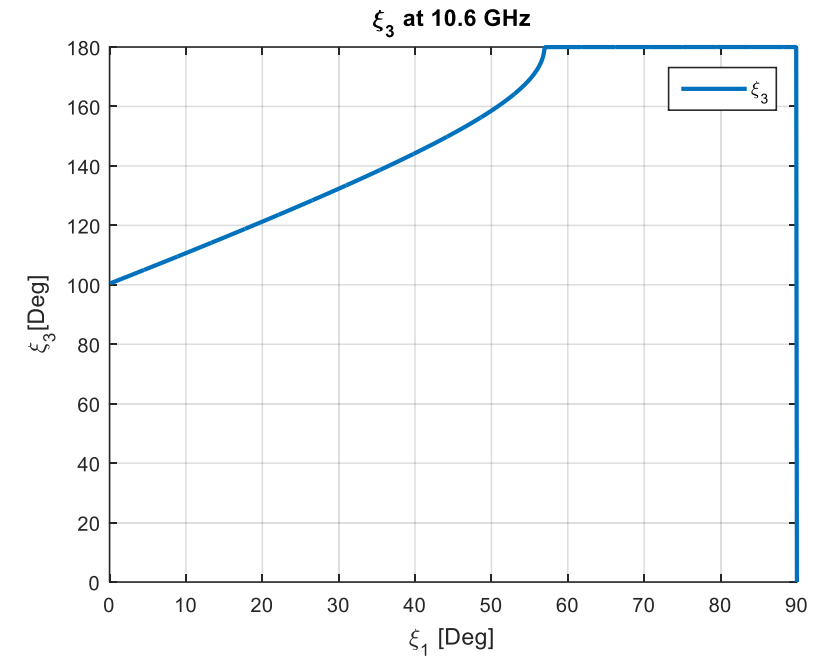
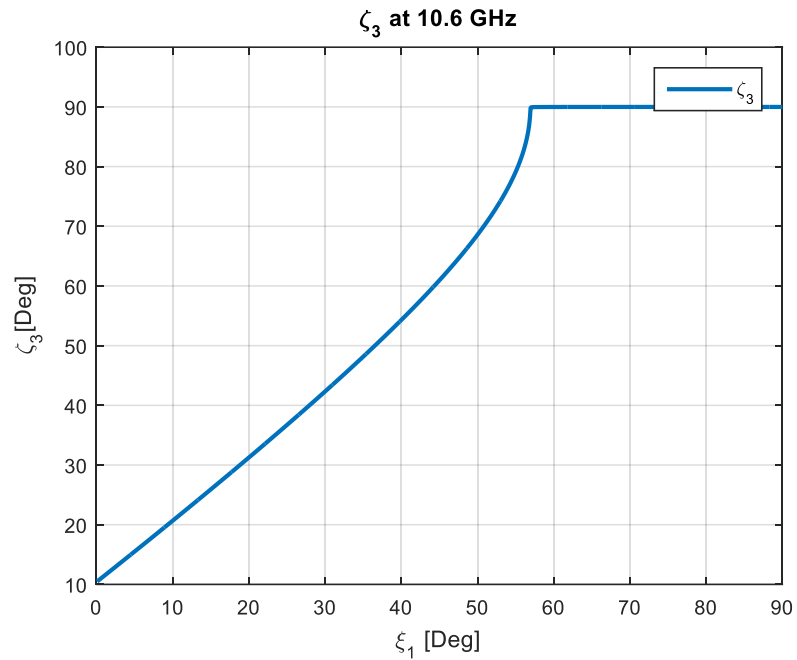
Material: FR4 [$\epsilon_r = 4.3$; $\tan \delta = 0.025$, at 10 GHz]

Prism angle: $\chi = 5^\circ$



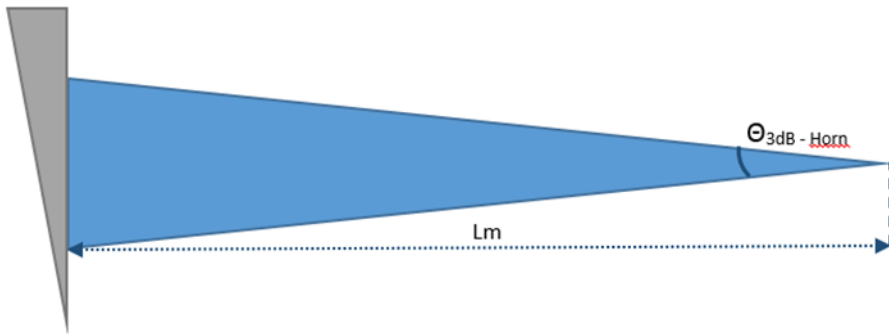
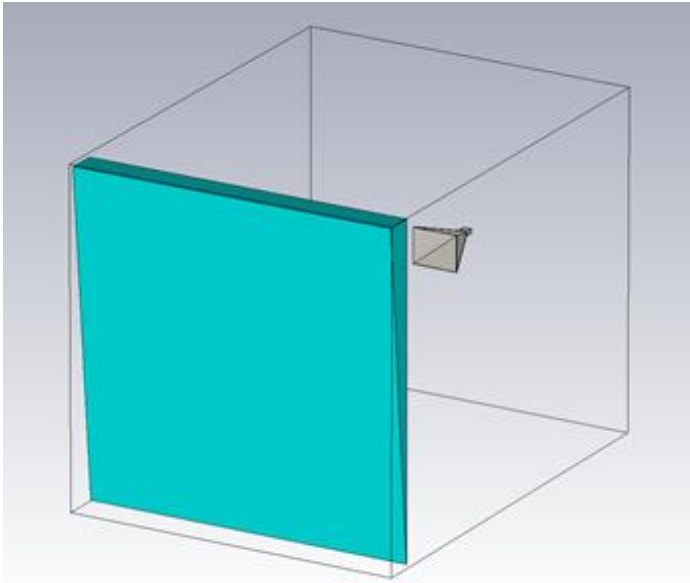
Step behavior!

Transmitted wave characterization (2)



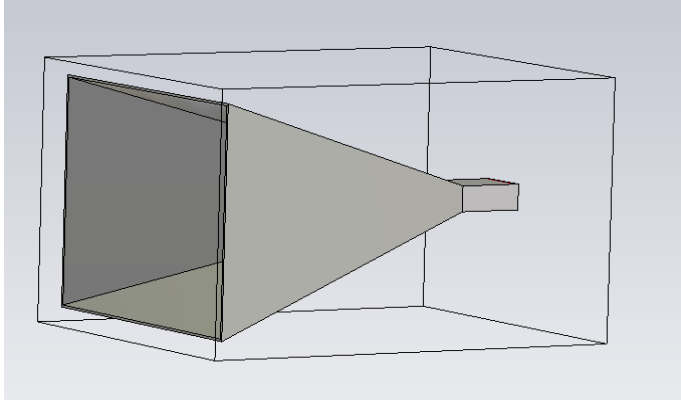
- Pointing direction of the phase vector and amplitude vector
- Step-behavior
- Critical angle \rightarrow Quasi-Surface Wave

Lossy-prism antenna

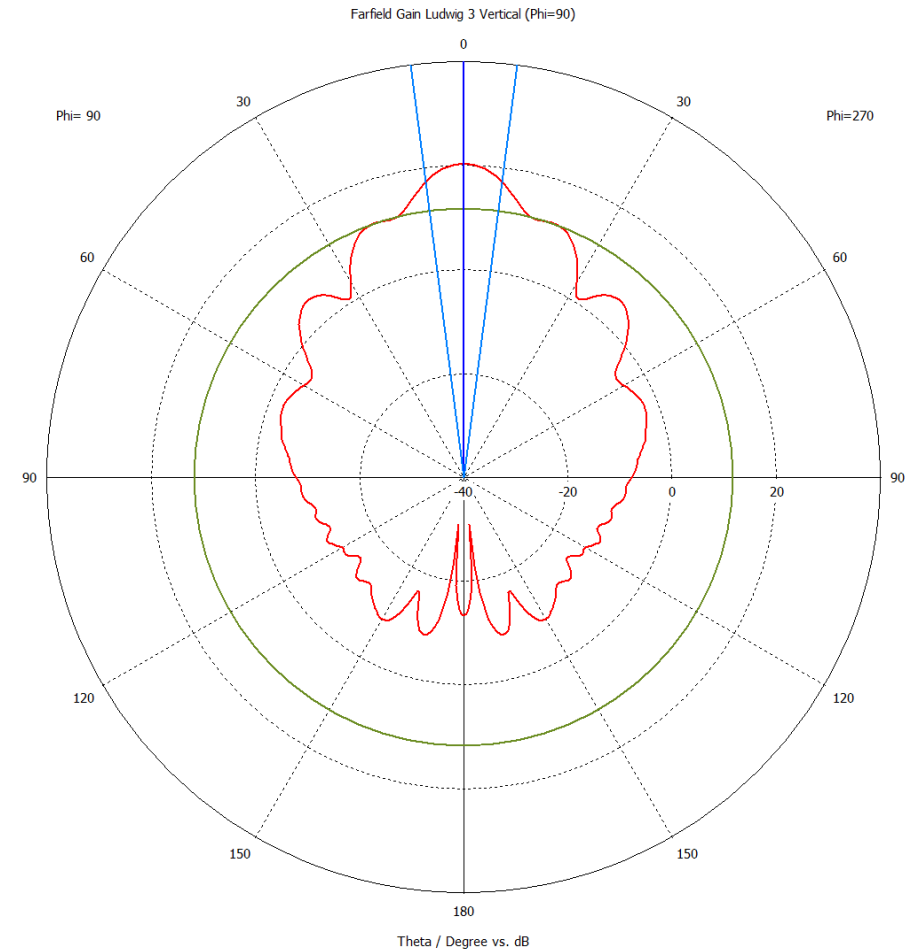


- System composed by the polarizer prism and a field source.
- The feed of the system is a Ku band rectangular tapered horn
- Distance between feed and prism:
- $r_{ff} \geq \frac{2D^2}{\lambda} \rightarrow r_{ff} = 1.0 \text{ m}$
- L_p takes into account the phase path between phase center and horn aperture.
- $L_m = r_{ff} + L_p$
- Prism Side $L = \sin \theta_{3dB} * (L_m + \frac{r_{ff}}{2})$
- The vertical side is 0.557 m long.

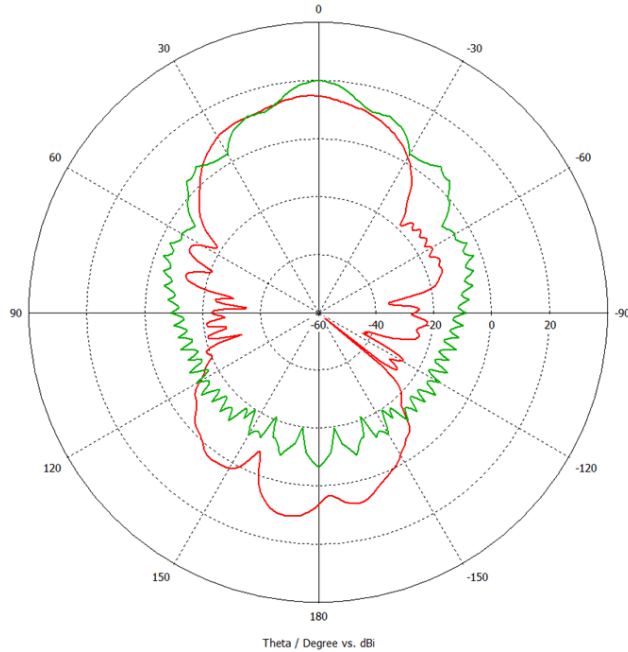
Full-wave analysis



- Ku band rectangular tapered horn [WR90]
- Lin.Pol.
- Op BW: $f_c = 10.6$ GHz.
- $D_{\max} = 20.1$ dB
- $\theta_{3dB} = 14.5$ deg
- Horn phase centre to provide a finite section quasi-plane wave radiated field at the prism interface.

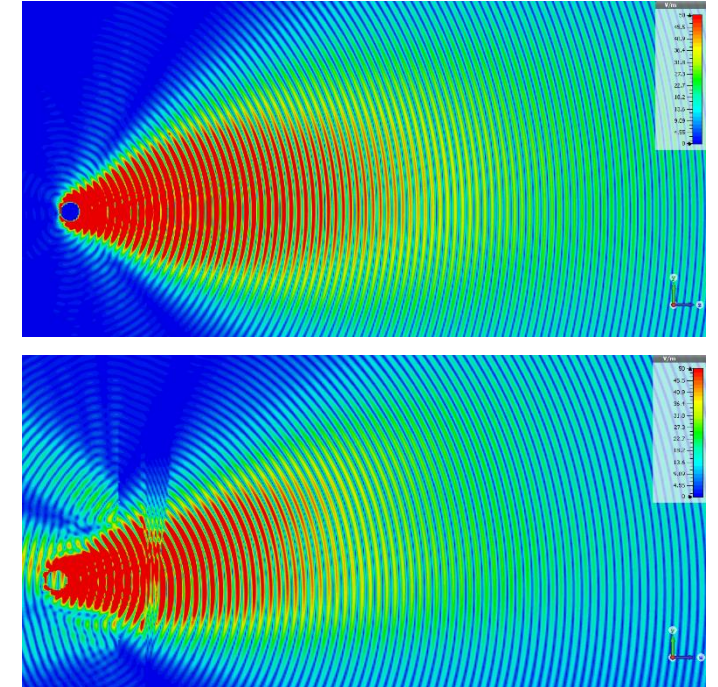


Full-wave analysis (2)



CST pattern : tapered horn (green) Vs.system horn+prism (red).

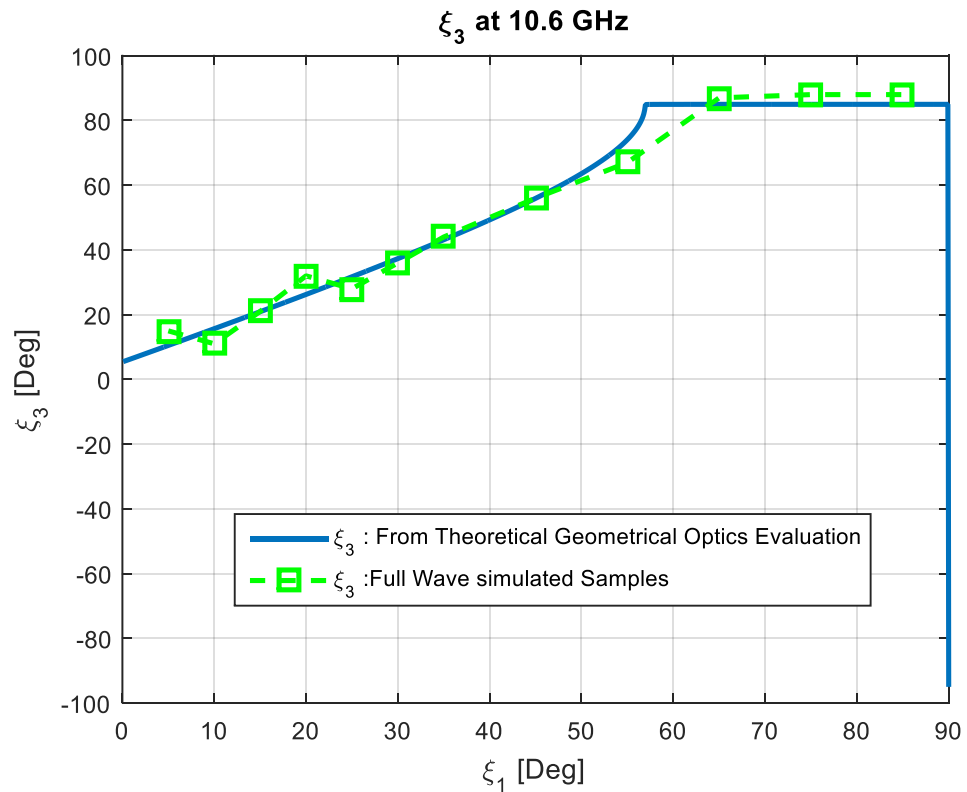
- Depointing in line with the theoretical analysis
- Defocusing due to prism multiple reflection
- Back lobes



Electric field along the propagation y-z plane with dielectric FR4 prism and incidence $\xi_1 = 5^\circ$.

Frequency	10.6 GHz	10.6 GHz
Directivity peak	20.1 dB	14.8 dB
Pointing Angle	0.0°	3°
θ_{3dB}	14.3°	29.2°
SLL	-8.3	-3.9

Validation



Very Good Accordance with theoretical values

TARGET	SINGLE HORN	HORN+PRISM ($\chi_{1i} 0^\circ$)	HORN+PRISM ($\chi_{1i} 5^\circ$)	HORN+PRISM ($\chi_{1i} 45^\circ$)
MAIN LOBE MAGNITUDE	20.1 dB	14.8 dB	14.4 dB	15.2 dB
MAIN LOBE DIRECTION	0.0°	3°	6°	57°
HPBW	14.3°	29.2°	37.8°	31.1°

A power problem

Up to now, we talked about the wave attenuation in the lossy medium

We considered an incident wave, carrying a certain power, we considered the transmitted wave, and we found the conditions under that the transmitted power is not attenuated.

However: how much power can we transmit?

We must consider the reflection coefficient!

Reflection coefficient

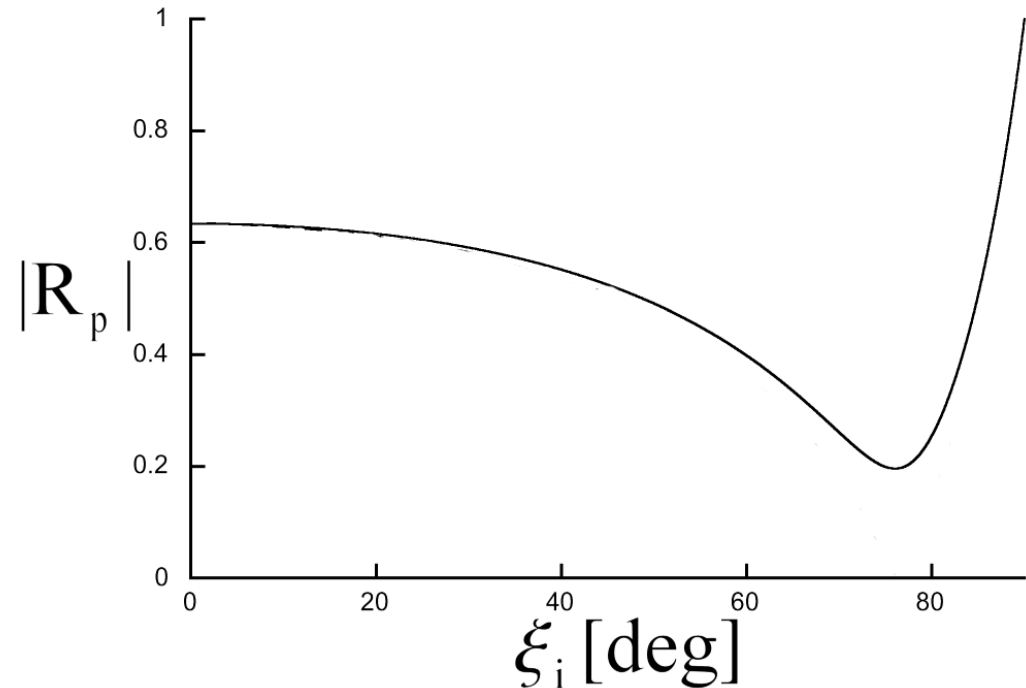
We need to maximize the transmitted power

It is possible to transmit the whole incident power?

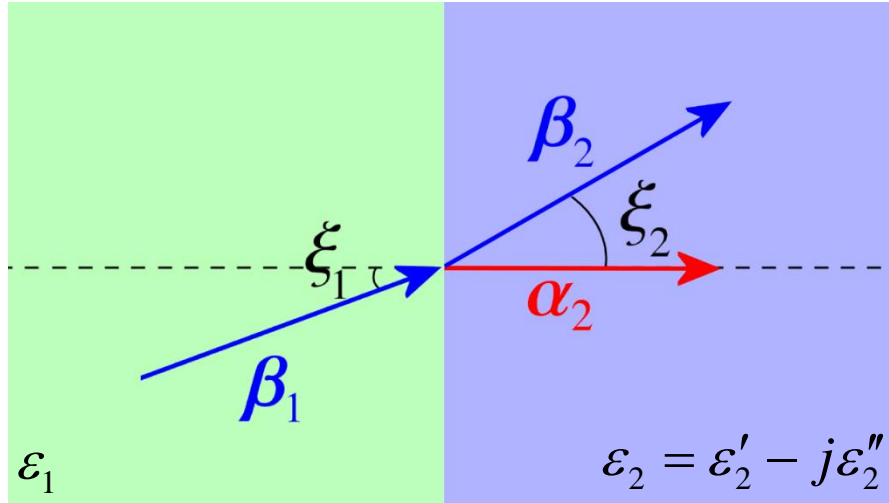
As it is well known in parallel polarization the total transmission can be obtained.

However, with lossy materials only a minimum of the reflection coefficient can be obtained.

The angle of the minimum reflection coefficient is called **Pseudo-Brewster angle**



Total Transmission



The expressions of the reflection coefficients:

$$R_E = \frac{k_{1x} - k_{2x}}{k_{1x} + k_{2x}} \quad \text{E-Polarization (s-polarization)}$$

$$R_H = \frac{\varepsilon_2 k_{1x} - \varepsilon_1 k_{2x}}{\varepsilon_2 k_{1x} + \varepsilon_1 k_{2x}} \quad \text{H-Polarization (p-polarization)}$$

If the incident wave is homogeneous, we have a difference between a real and a complex quantity,

It cannot be zero

However, if the incident wave is inhomogeneous, then the reflection coefficient can be zero!

Total Transmission (2)

As it is well known, total transmission can be obtained only in H-polarization

$$R_H = \frac{\varepsilon_2 k_{1x} - \varepsilon_1 k_{2x}}{\varepsilon_2 k_{1x} + \varepsilon_1 k_{2x}}$$

$$\Downarrow = 0$$

$$\varepsilon_2 k_{1x} = \varepsilon_1 k_{2x}$$

$$\Downarrow$$

$$k_{1y} = k_0 \sqrt{\frac{\varepsilon_1 \varepsilon_2}{\varepsilon_2 + \varepsilon_1}} = k_0 \gamma$$

Total transmission
condition

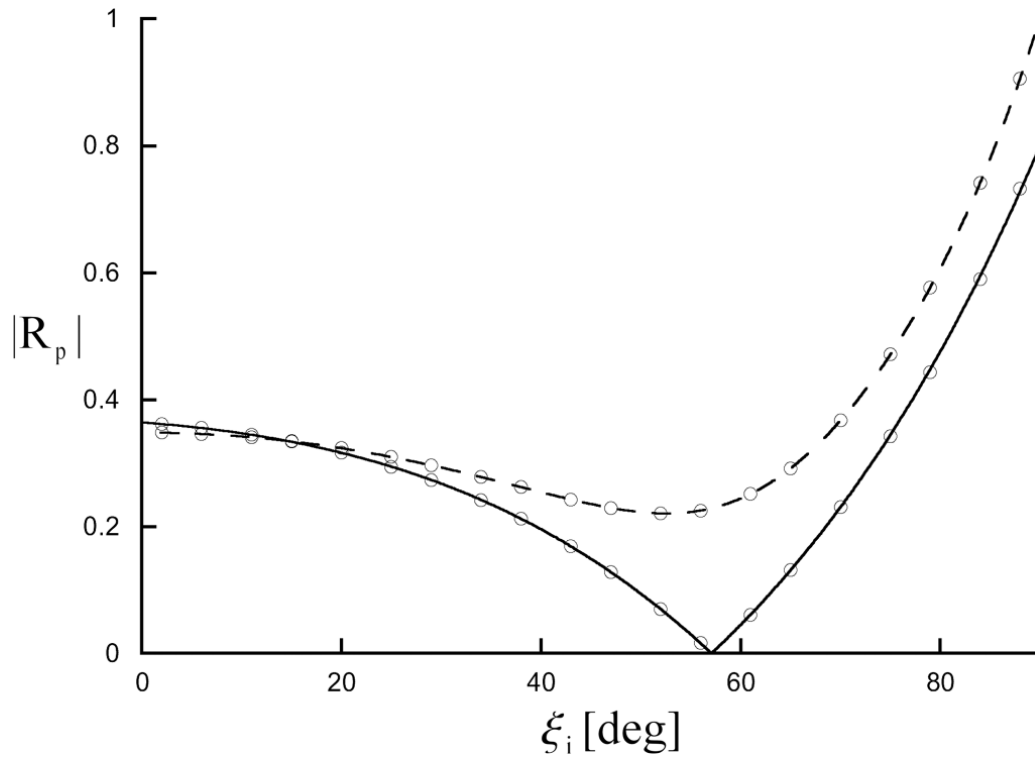
Total Transmission (3)

Imposing the dispersion equation, we can obtain the conditions on the properties of the incident wave

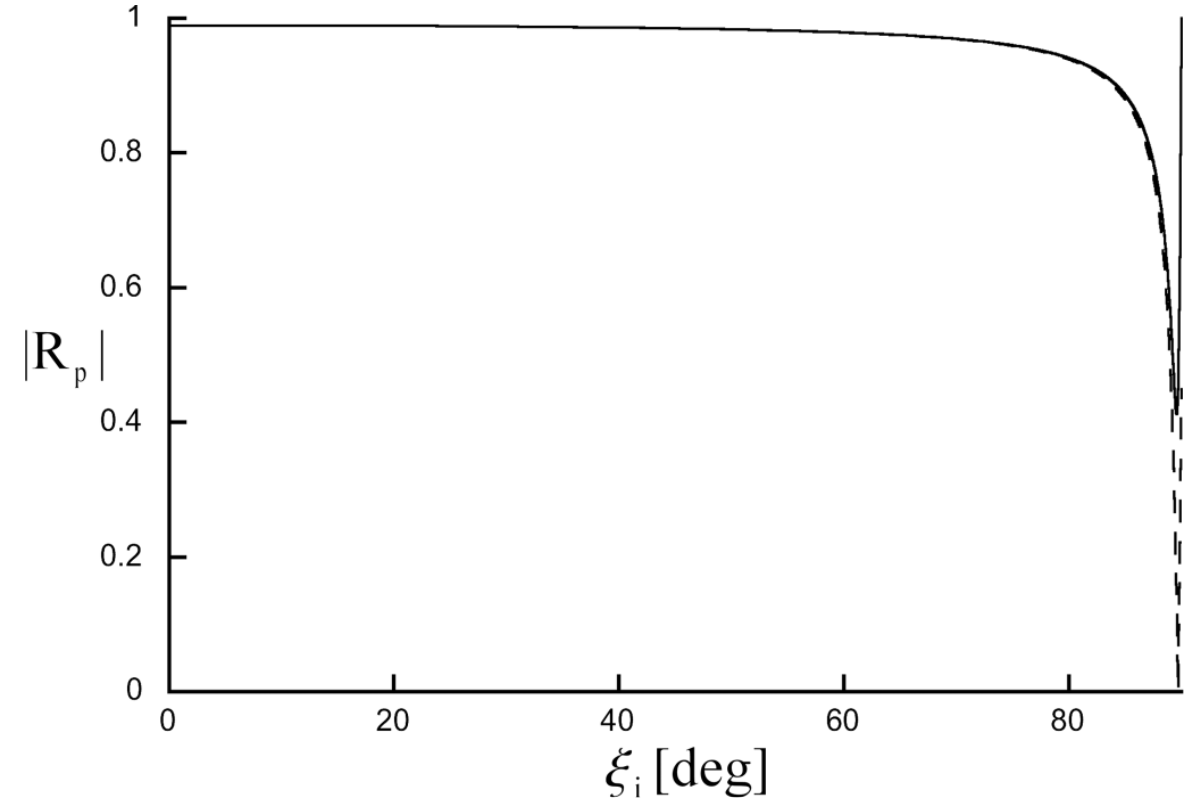
$$\tan \xi_B = \frac{\alpha_B}{\beta_B} \cot \phi_\gamma \quad \text{Generalized Brewster angle}$$

$$\beta_B = k_0 \sqrt{\frac{\varepsilon_1 + |\gamma|^2}{2}} \sqrt{1 + \sqrt{1 - \varepsilon_1 \left(\frac{2\gamma'}{\varepsilon_1 + |\gamma|^2} \right)^2}} \quad \text{Brewster phase amplitude}$$

Total Transmission (4)

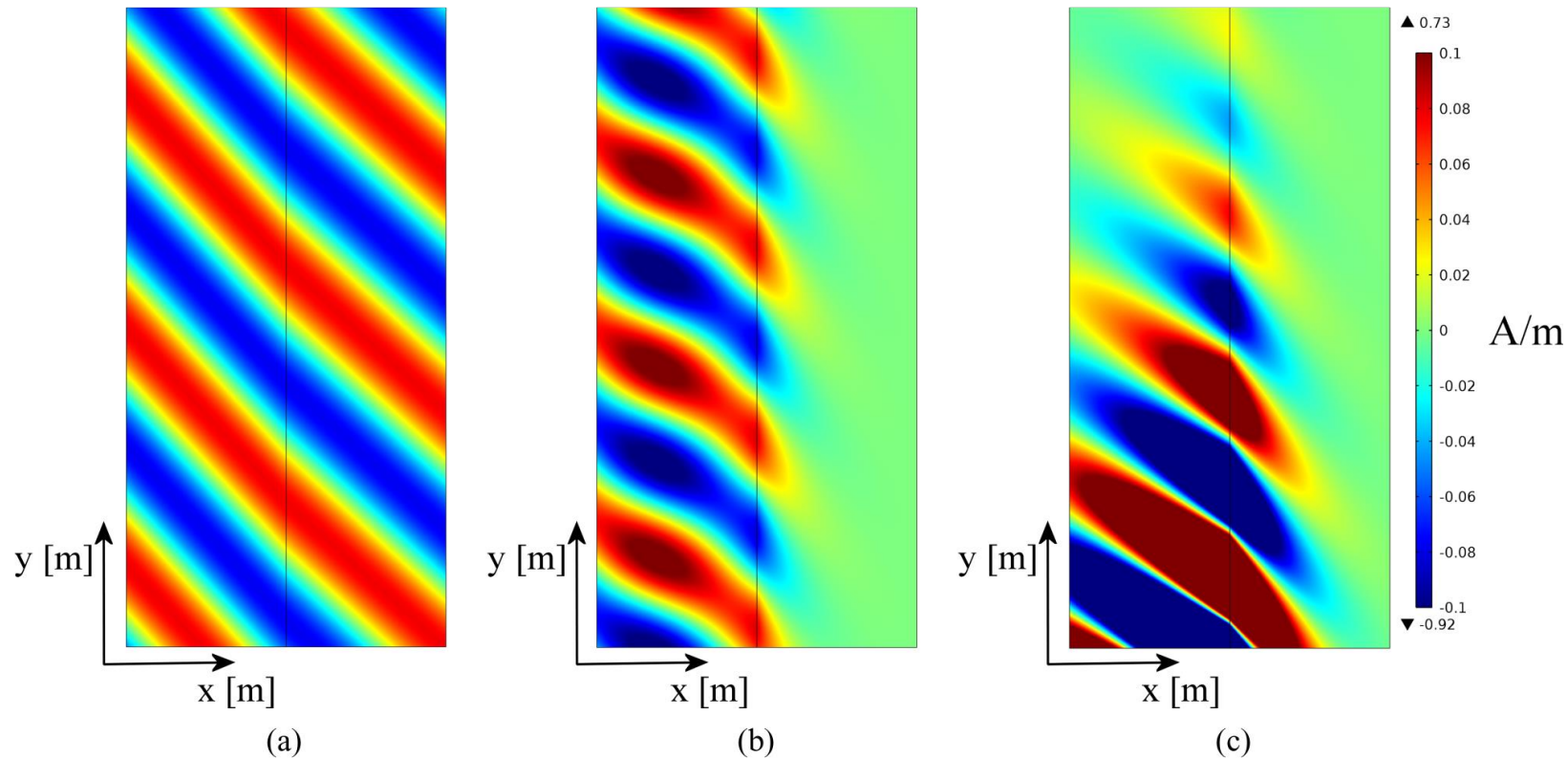


Reflection coefficient for an interface between air and gold at $\lambda = 7.7 \mu\text{m}$



Reflection coefficient for an interface between air and seawater at a frequency of 100MHz

Total Transmission (5)



Two-dimensional maps of the real part of the magnetic field at the interface between a vacuum and medium 2 with relative permittivity ϵ_2 .

- (a) Medium 2 is lossless, with $\epsilon_2 = 0.84$, the incident wave is homogeneous, and the incident angle is the Brewster angle.
- (b) Medium 2 is dissipative, with $\epsilon_2 = 0.84 + i1.91$ (gold at wavelength $\lambda = 7.7 \mu\text{m}$), the incident wave is homogeneous, and the incident angle is the pseudo-Brewster angle.
- (c) Medium 2 is dissipative [as in (b)], the incident wave is inhomogeneous with $\beta_1 = \beta_B$, and the incident angle is the Brewster angle.

Conclusions

- We investigated the fundamental differences between homogeneous and inhomogeneous waves
- The direction of attenuation in a lossy media depends on the nature of the incident wave, if it is inhomogeneous, the direction of the attenuation can be driven
- Under specific conditions it is possible to make the attenuation parallel to the interface, i.e., to deeply penetrate the dissipative material
- We presented a practical realization of the DPW through a LWA.
- The comparisons between the penetration depths of the LWA and a canonical Horn antenna have been shown.
- An alternative approach to obtain the DPW has been proposed through a Lossy-prism antenna
- The possibility to obtain total transmission of the incident power in a lossy medium is still possible if the incident wave is inhomogeneous

Inhomogeneous waves seem to have several interesting features!