Frequency domain analysis (of the GPR transmitted field)

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- Analytical solutions for the wire antenna in the FD
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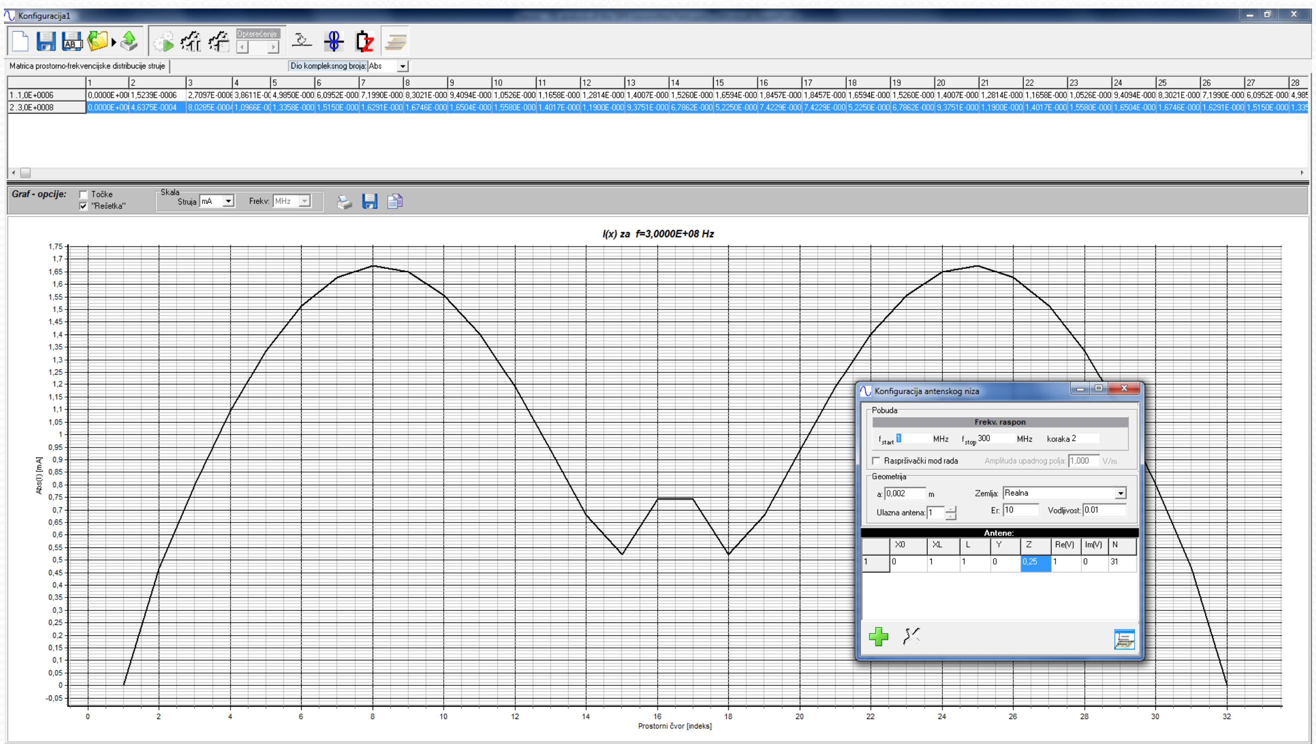
SuzanaFD

Introduction

SuzanaFD

- Introduction to software SuzanaFD



Integral equations

…and methods for their solution

Pocklington equation (1897.)

Frequency domain

$$E_x^{exc}(\omega) = -\frac{1}{j4\pi\omega\varepsilon_{eff}} \int_0^L \left(\frac{\partial^2}{\partial x^2} - \gamma^2 \right) I(x',\omega)\, g(x,x')\, dx'$$

Time domain

$$\left(\varepsilon \frac{\partial}{\partial t} + \sigma \right) E_x^{exc}(t) = -\left(\frac{\partial^2}{\partial x^2} - \mu\sigma \frac{\partial}{\partial t} - \mu\varepsilon \frac{\partial^2}{\partial t^2} \right) \int_0^L \frac{I\left(x', t - \frac{R}{v} \right)}{4\pi R}\, dx'$$

Hallen equation (1938.)

Frequency domain

$$\int_0^L I(x',\omega)\, g(x,x')\, dx' = C\cos(kx) + B\sin(kx) - \frac{j4\pi}{Z_0}\int_0^L E_x^{exc}(x',\omega)\sin\left[k(x-x')\right] dx'$$

Time domain

$$\int_0^L \frac{I\left(x',t-\frac{R}{c}\right)}{4\pi R}\, dx' = F_0\left(t-\frac{x}{c}\right) + F_L\left(t-\frac{L-x}{c}\right) + \frac{1}{2Z_0}\int_0^L E_x^{exc}\left(x',t-\frac{|x-x'|}{c}\right) dx'$$

Methods of solving integral equations

- Numerical solutions
  - Complex geometries
  - Wide range of parameters
- Analytical solutions
  - Canonical geometries
  - Limited parameters
  - Use of approximations
  - Procedure control
  - Simple solutions

Analytical solution

Frequency domain approximations and procedure

Frequency domain approximations – thin-wire approximation

$$\vec{A}(\vec{r},\omega)=\frac{\mu}{4\pi}\int_{V'}\vec{J}(\vec{r}\,')g(\vec{r},\vec{r}\,')dV' \qquad A_x(x,\omega)=\frac{\mu}{4\pi}\int_{0}^{L}I(x')g(x,x')dx'$$

$$\vec{E}=-\nabla\varphi-j\omega\vec{A} \qquad E_x=-\frac{\partial\varphi}{\partial x}-j\omega A_x$$

Frequency domain approximations – reflection coefficient

$$U_{11} = 2\int_0^\infty \frac{e^{-2\mu_1(z+h)}}{\mu_1+\mu_2} J_0(\lambda\rho)\lambda d\lambda$$

$$W_{11} = 2\int_0^\infty \frac{(\mu_1-\mu_2)e^{-2\mu_1(z+h)}}{k_2^2\mu_1+k_1^2\mu_2} J_0(\lambda\rho)\lambda d\lambda$$

Sommerfeld integrals

$$\Gamma_{ref} = \frac{\dfrac{1}{\varepsilon_r+\dfrac{\sigma}{j\omega\varepsilon_0}}\cos\theta - \sqrt{\dfrac{1}{\varepsilon_r+\dfrac{\sigma}{j\omega\varepsilon_0}}-\sin^2\theta}}{\dfrac{1}{\varepsilon_r+\dfrac{\sigma}{j\omega\varepsilon_0}}\cos\theta + \sqrt{\dfrac{1}{\varepsilon_r+\dfrac{\sigma}{j\omega\varepsilon_0}}-\sin^2\theta}}$$

Fresnel reflection coefficient approach

Frequency domain approximations – integral function

$$\int_0^L I(x')\,g(x,x')\,dx' = \int_0^L I(x)\,g(x,x')\,dx' + \int_0^L \left[I(x') - I(x)\right] g(x,x')\,dx'$$

$$I(x') - I(x) \equiv 0$$

$$\int_0^L I(x')\,g(x,x')\,dx' = I(x)\int_0^L g(x,x')\,dx'$$

Green's function

$$\int_0^L g(x,x')\,dx' = \begin{cases} 2\ln\dfrac{L}{a} \\ 2\left(\ln\dfrac{L}{a} - \Gamma_0^{ref}\ln\dfrac{L}{2h}\right) \\ 2\left(\ln\dfrac{L}{a} - \Gamma_E^{ref}\ln\dfrac{L}{2d}\right) \end{cases}$$

Free space

Above lossy ground

Below lossy ground

$$\frac{\partial^2 I(x,\omega)}{\partial x^2} - \gamma^2 I(x,\omega) = j\frac{4\pi\gamma^2}{\omega\mu\Psi(\omega)}E_x(\omega)$$

Differential equation

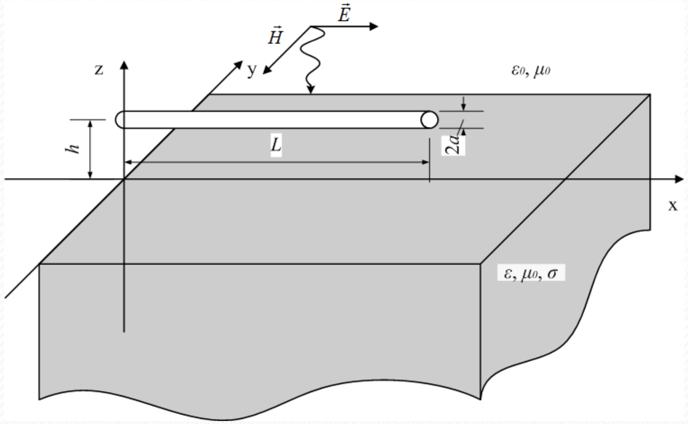
Induced current

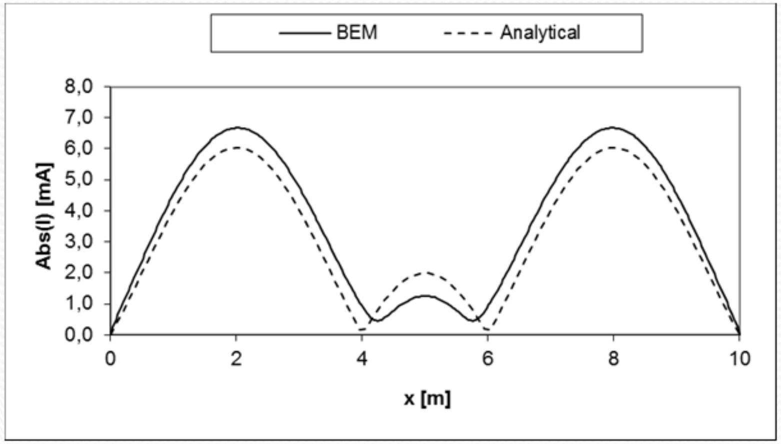
$$I(x,\omega)=\frac{4\pi}{j\omega\mu\Psi(\omega)}E_x(\omega)\left[1-\frac{\cosh\left(\gamma\left(\frac{L}{2}-x\right)\right)}{\cosh\left(\gamma\frac{L}{2}\right)}\right]$$

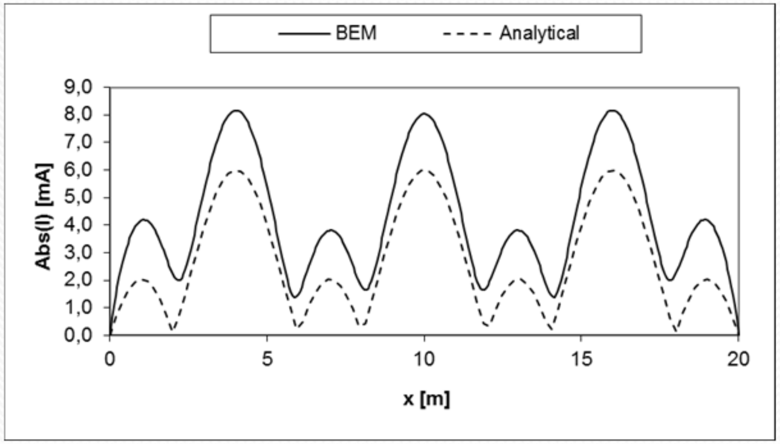
Analytical solutions

Examples

Wire above ground (power cable…)



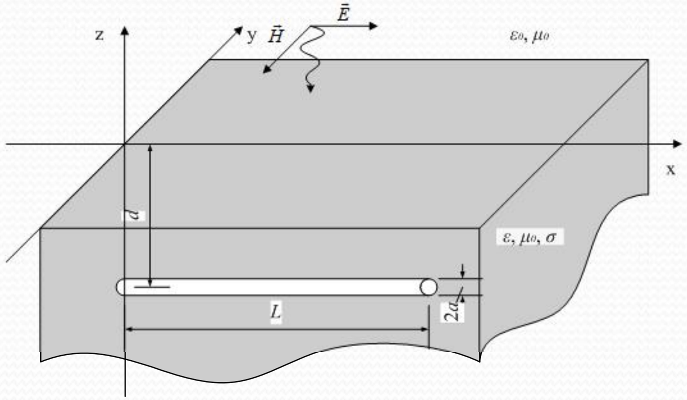


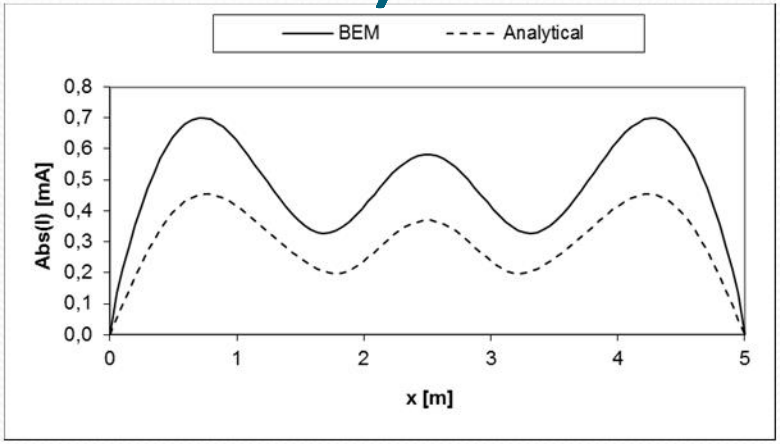


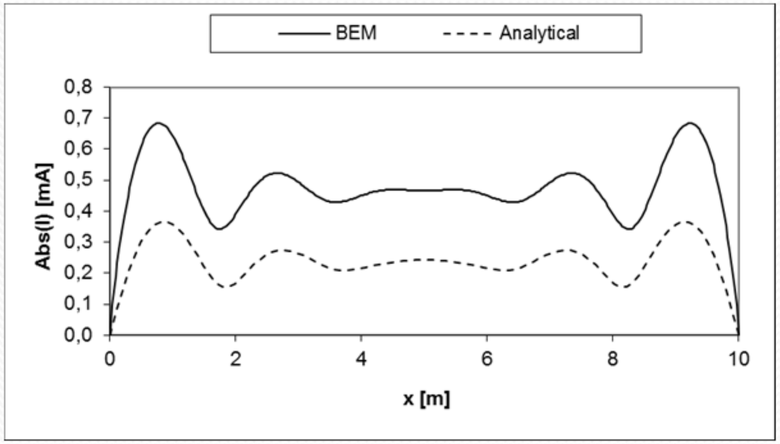
$$I(x,\omega) = \frac{4\pi}{j\omega\mu\Psi(\omega)} E_x^{exc}(\omega) \left[1 - \frac{\cos\left(k\left(\frac{L}{2} - x\right)\right)}{\cos\left(k\frac{L}{2}\right)} \right]$$

f=50 MHz

Wire below ground (telecommunication cable…)





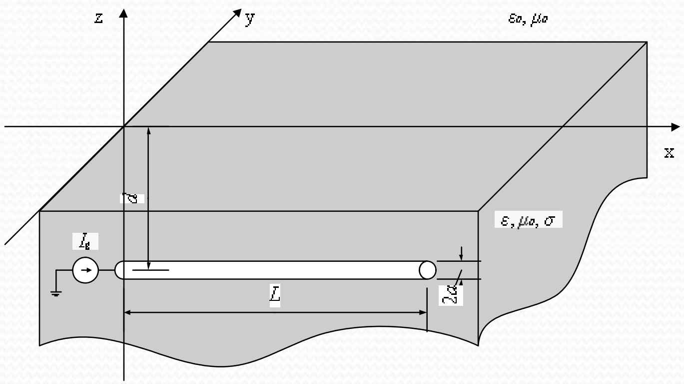


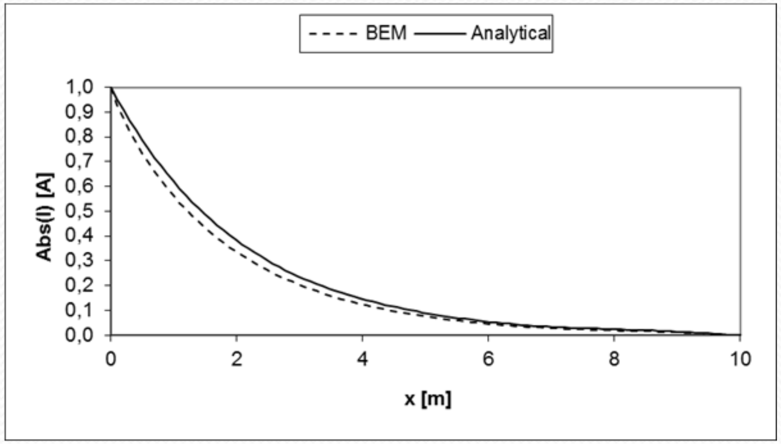
$$I(x,\omega) = \frac{4\pi e^{j\frac{a}{v}\omega}}{j\omega\mu\Psi(\omega)} E_x^{exc}(\omega)\left[1 - \frac{\cosh\left(\gamma\left(\frac{L}{2} - x\right)\right)}{\cosh\left(\gamma\frac{L}{2}\right)}\right]$$

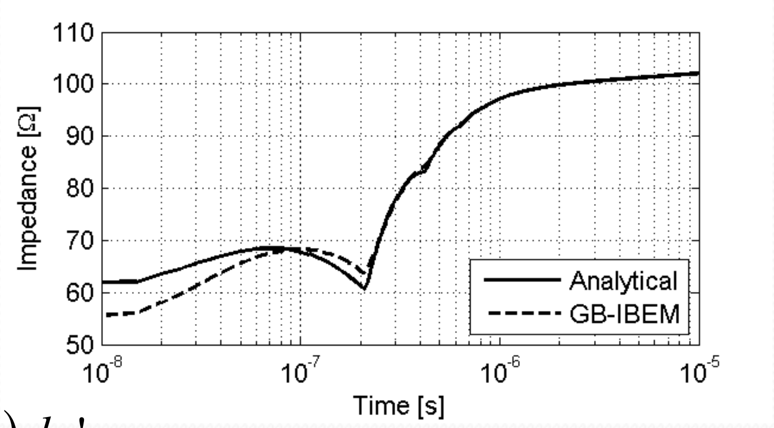
f=50 MHz

Wire below ground (grounding electrode...)

f=10 MHz





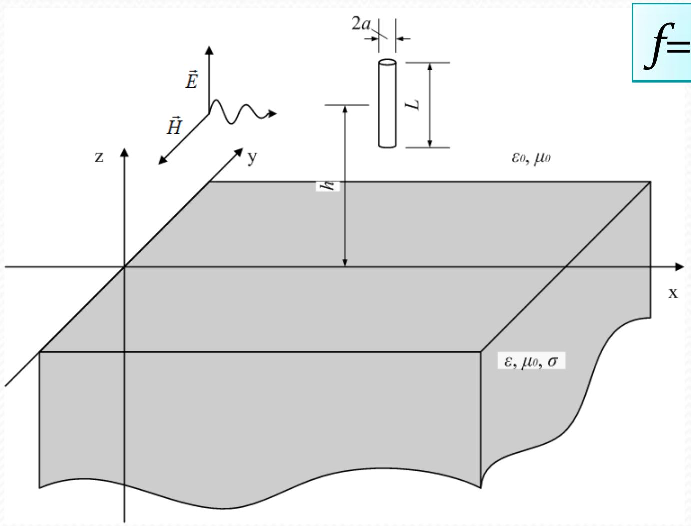


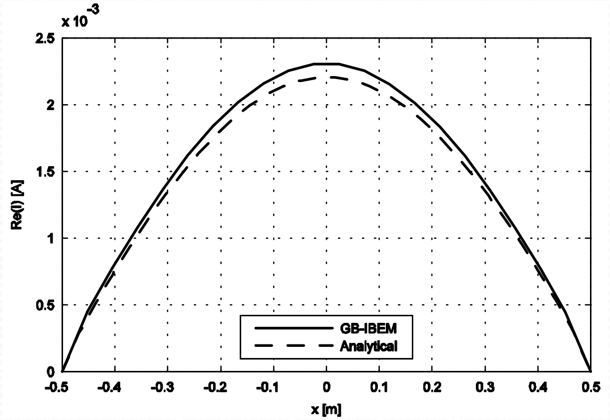
$$I(x,\omega) = I_g(\omega)\frac{\sinh\left[\gamma(L-x)\right]}{\sinh(\gamma L)}$$

$$V^{sct}(x,\omega) = \frac{\gamma I_g(\omega)}{j4\pi\omega\varepsilon_{eff}\sinh(\gamma L)}\int_0^L \cosh\left[\gamma(L-x)\right]g(x,x')dx'$$

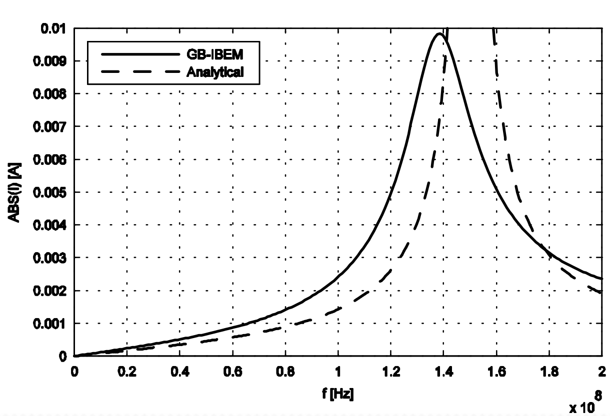
Vertical wire above ground (receiving antenna…)

f=168.2 MHz





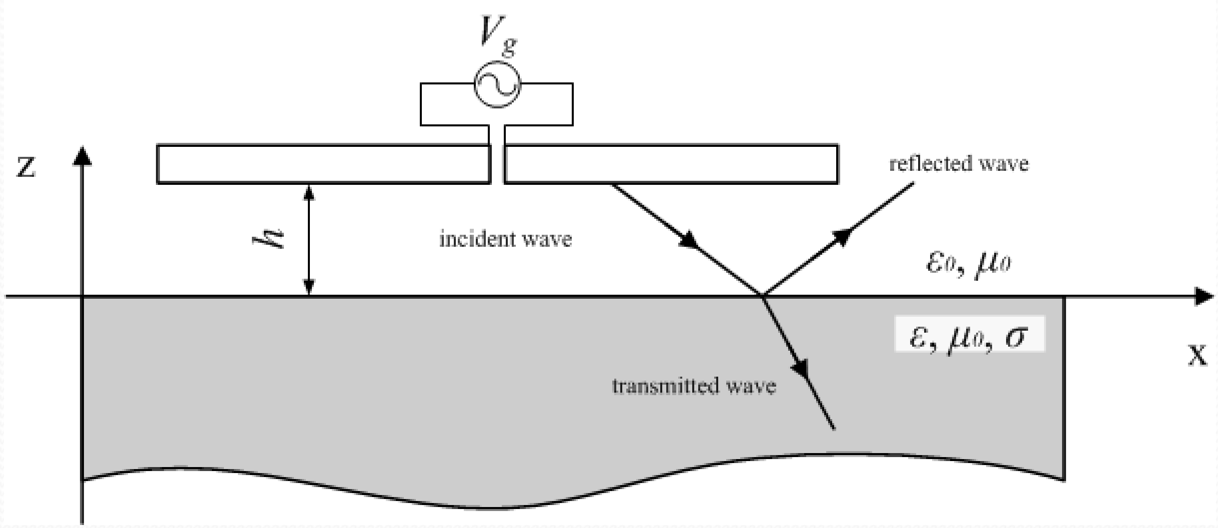
$$I(z,\omega)=\frac{4\pi}{j\omega\mu\Psi(z,\omega)}E_z^{exc}(\omega)\left[1-\frac{\cos(k(h-z))}{\cos\left(k\frac{L}{2}\right)}\right]$$



Šesnić: Frequency domain analysis (of the GPR transmitted field)

Transmitted field - formulation

Transmitting GPR dipole antenna



Pocklington integral equation

$$E_x^{exc}(\omega) = j\omega \frac{\mu}{4\pi} \int_0^L I(x',\omega)\, g(x,x')\, dx'$$

$$-\frac{1}{j4\pi\omega\varepsilon} \frac{\partial}{\partial x} \int_0^L \frac{\partial I(x',\omega)}{\partial x'}\, g(x,x')\, dx'$$

Transmitted field in the lower medium

$$E_x^{tr}(x,z,\omega) = -\frac{1}{j4\pi\omega\varepsilon_{eff}}\left[\int_0^L \frac{\partial I(x',\omega)}{\partial x'}\frac{\partial G(x,x',z)}{\partial x'}dx' + \gamma^2\int_0^L I(x',\omega)G(x,x',z)\,dx'\right]$$

$$E_z^{tr}(x,z,\omega) = \frac{1}{j4\pi\omega\varepsilon_{eff}}\int_0^L \frac{\partial I(x',\omega)}{\partial x'}\frac{\partial G(x,x',z)}{\partial z}dx'$$

Transmitted field in the lower medium

$$G(x,x',z)=\Gamma_{tr}^{MIT} g_E(x,x',z)$$

Green's function

$$\Gamma_{tr}^{MIT}=\frac{2\varepsilon_{eff}}{\varepsilon_{eff}+\varepsilon_0} \qquad \varepsilon_{eff}=\varepsilon_r\varepsilon_0-j\frac{\sigma}{\omega}$$

$$g_E(x,x',z)=\frac{e^{-\gamma R}}{R} \qquad \gamma=j\omega\sqrt{\mu\varepsilon_{eff}}$$

Boundary element formalism

$$I(x') = I_{1i}\frac{x_{2i} - x'}{\Delta x} + I_{2i}\frac{x' - x_{1i}}{\Delta x}$$

$$\frac{\partial I(x')}{\partial x'} = \frac{I_{2i} - I_{1i}}{\Delta x}$$

Induced current

Boundary element formalism

$$E_x^{tr}(x,z,\omega) = -\frac{1}{j4\pi\omega\varepsilon_{eff}} \sum_{i=1}^{N_j} \left[\frac{I_{2i}-I_{1i}}{\Delta x} \int_{x_{1i}}^{x_{2i}} \frac{\partial G(x,x',z)}{\partial x'} dx' \right.$$

$$\left. +\gamma^2 \int_{x_{1i}}^{x_{2i}} \left[I_{1i} \frac{x_{2i}-x'}{\Delta x} + I_{2i} \frac{x'-x_{1i}}{\Delta x} \right] G(x,x',z)\, dx' \right]$$

$$E_z^{tr}(x,z,\omega) = \frac{1}{j4\pi\omega\varepsilon_{eff}} \sum_{i=1}^{N_j} \frac{I_{2i}-I_{1i}}{\Delta x} \int_{x_{1i}}^{x_{2i}} \frac{\partial G(x,x',z)}{\partial z} dx'$$

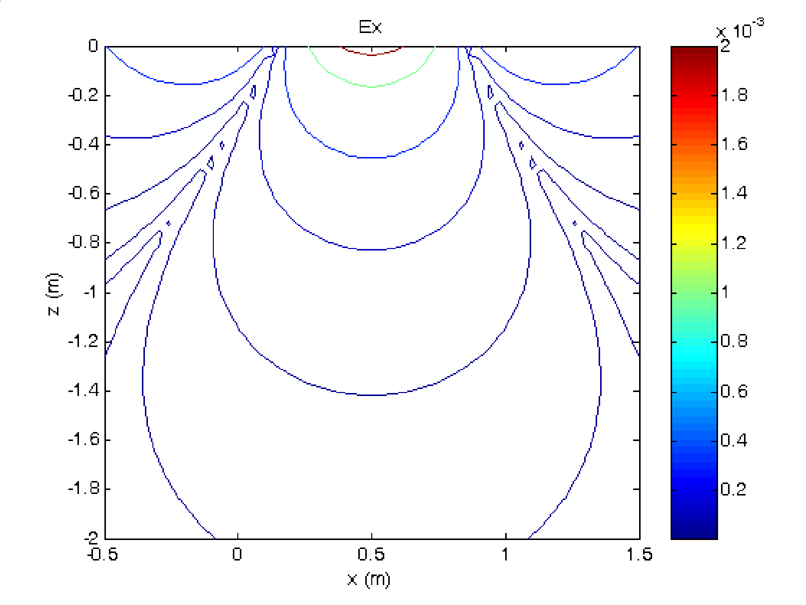
Numerical results

Parameters of the calculation

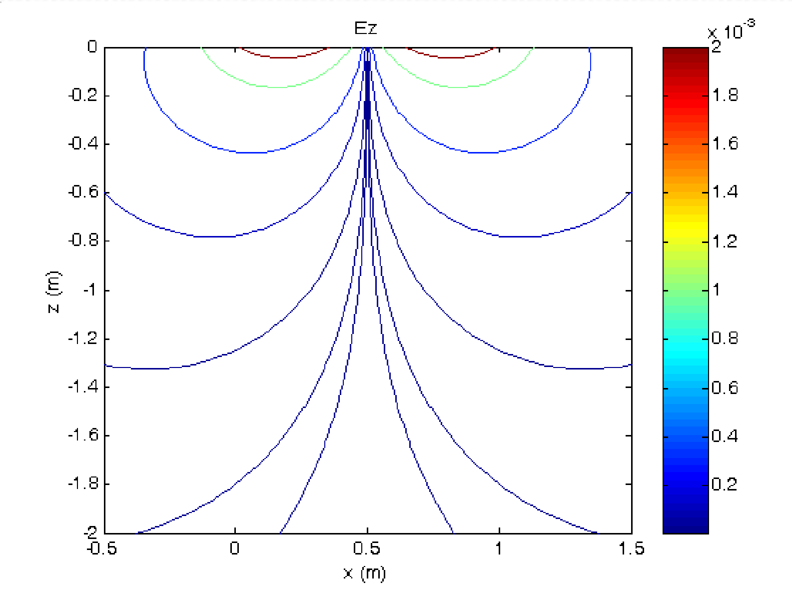
- Properties of the antenna
  - Length L=1 m
  - Radius a=2 mm
  - Height h=0.25 m
- Properties of the lower medium
  - Relative permittivity ε_r=10
  - Conductivity σ=10 mS/m
- Properties of the excitation
  - V_0=1 V, f=1, 10, 100, 300 MHz

Numerical results, *f*=1 MHz

E_x

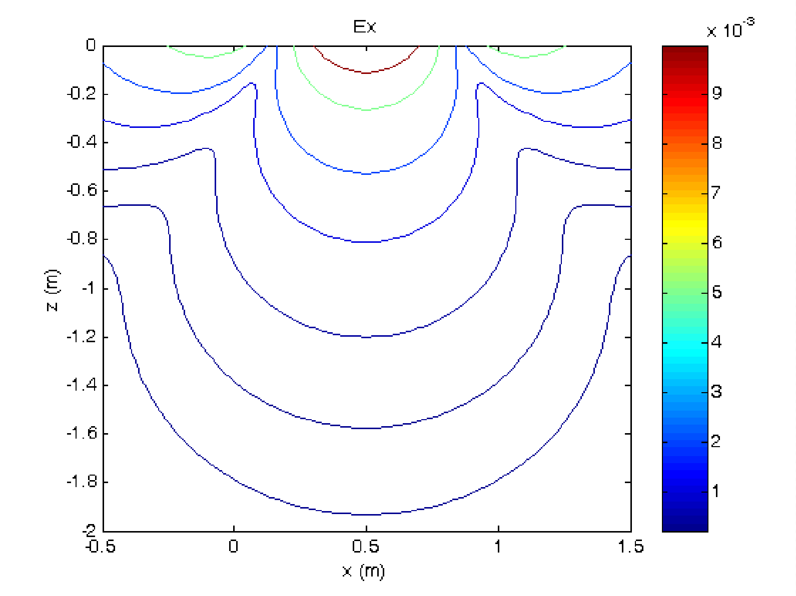


E_z

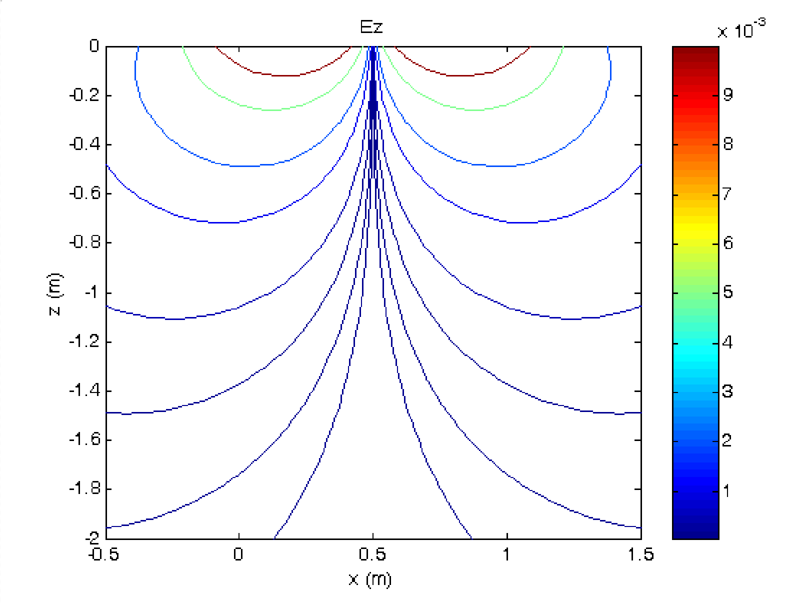


Numerical results, *f*=10 MHz

E_x

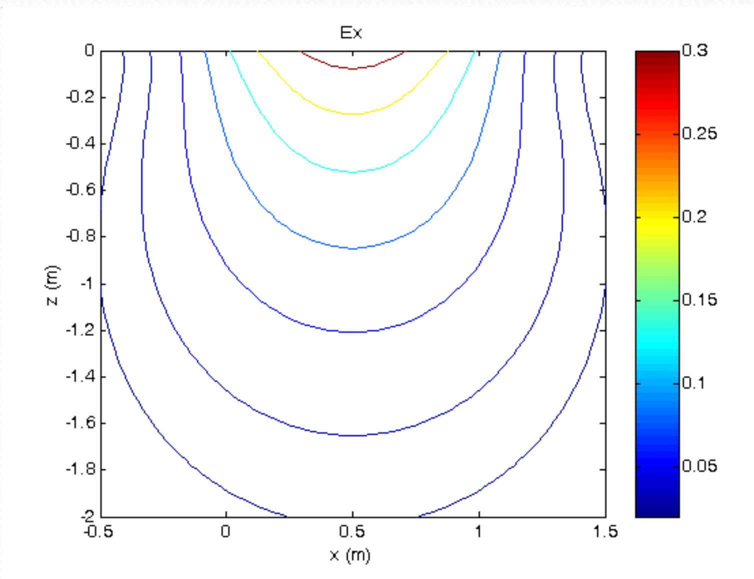


E_z

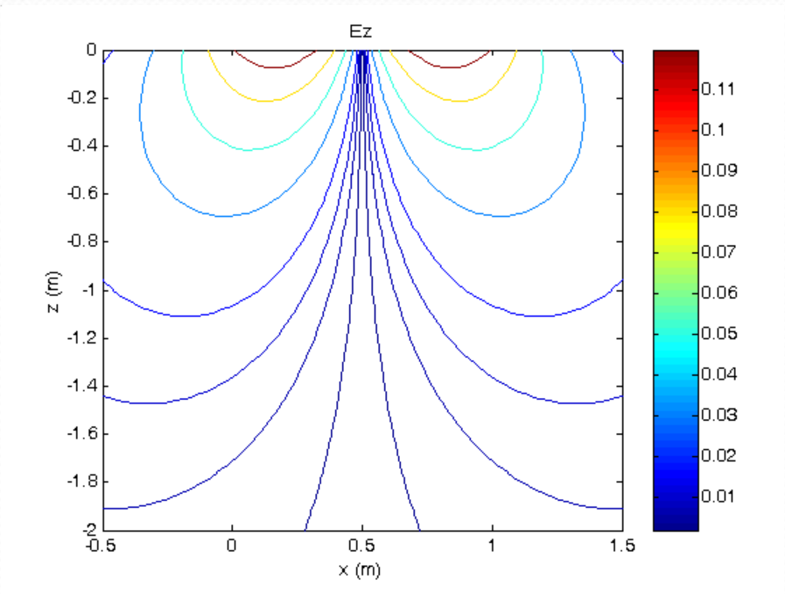


Numerical results, *f*=100 MHz

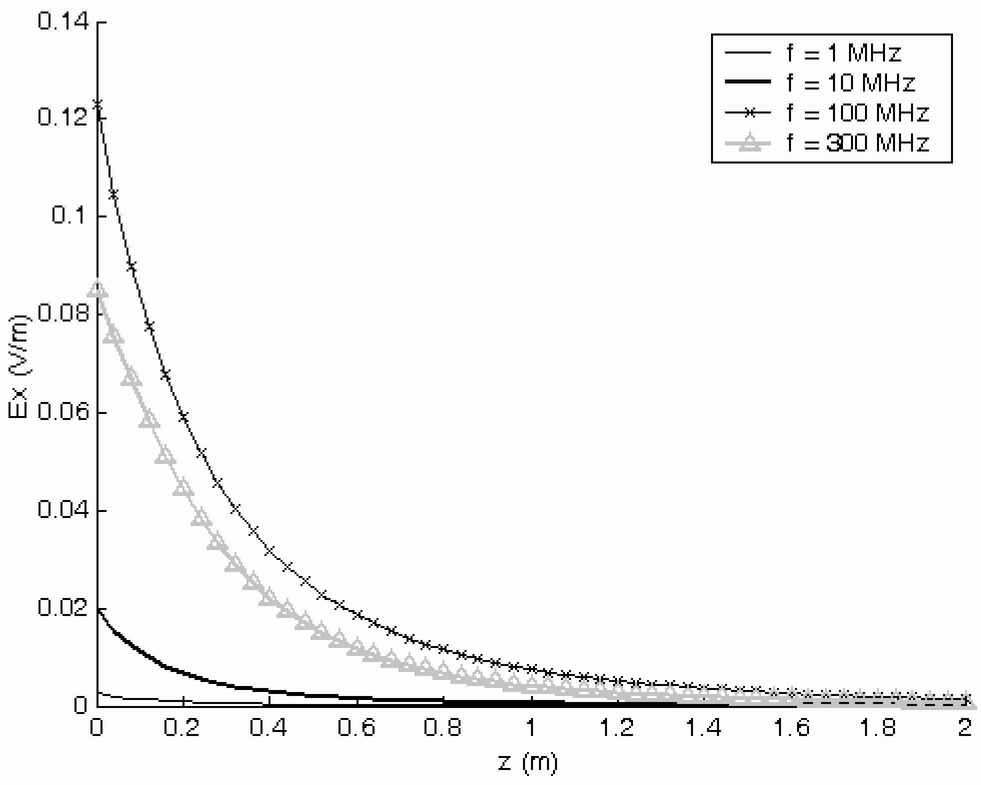
E_x



E_z



Numerical results, broadside field



Concluding remarks

Concluding remarks

- Software SuzanaFD, freely available
- Analytical solutions for the wire antenna in the FD
  - Future work – solution for the transmitting antenna
- Transmitted electrical filed in the FD

Thank you for your attention!