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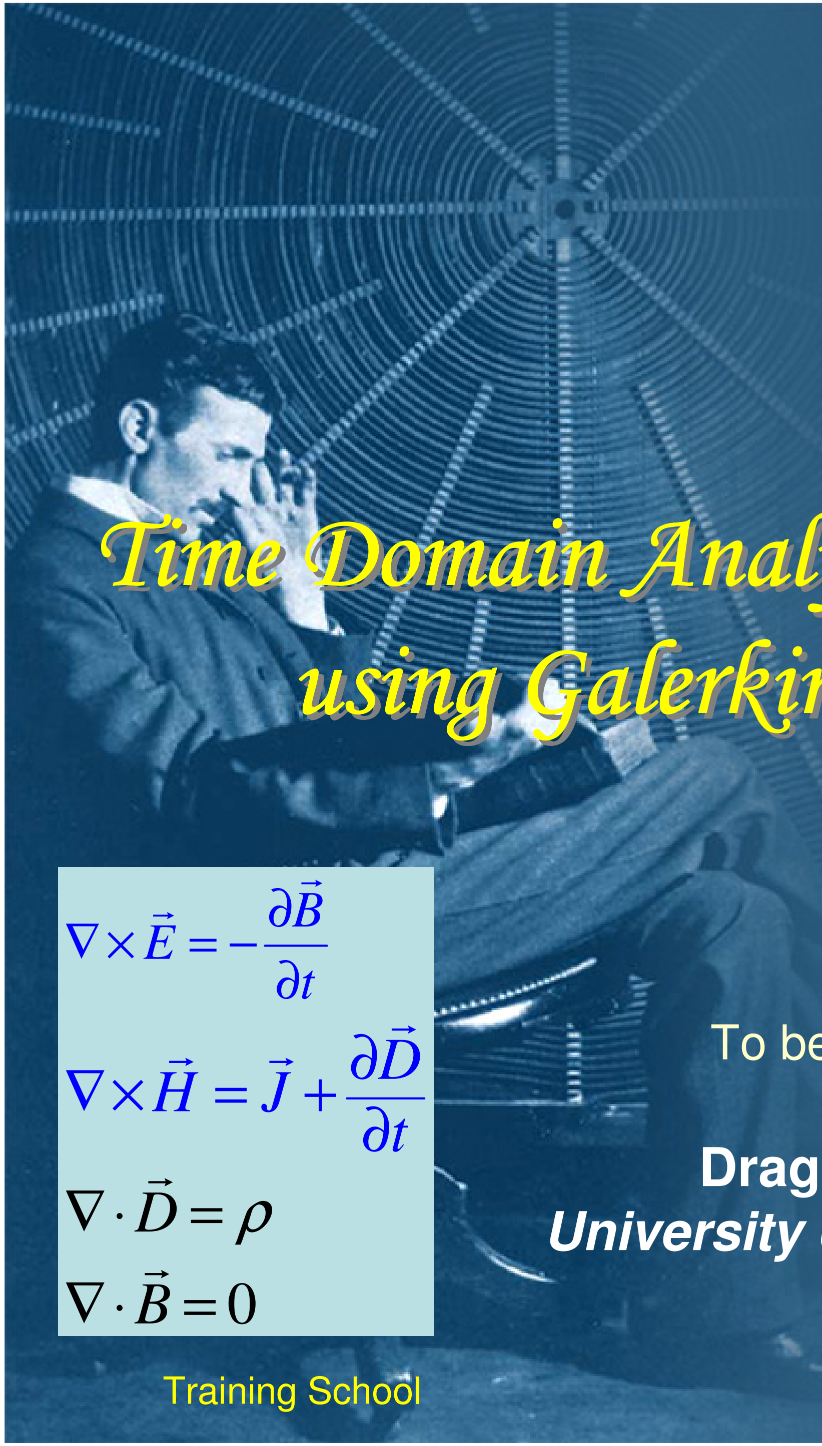
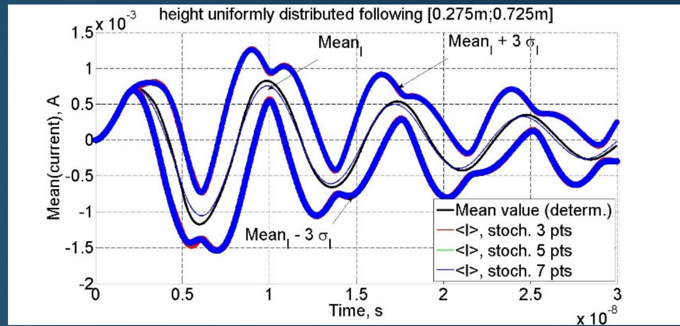




Time Domain Analysis of GPR Dipole Antenna using Galerkin-Bubnov Indirect Boundary Element Method

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

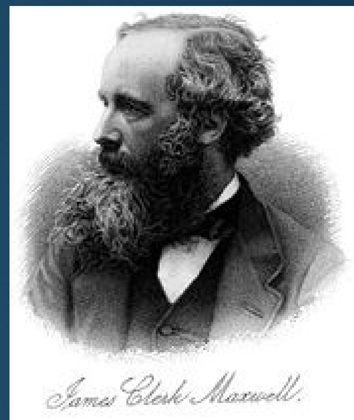
$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\nabla \cdot \vec{D} = \rho$$

$$\nabla \cdot \vec{B} = 0$$

To be presented by

Dragan Poljak
University of Split, Croatia



Training School

Split, 8 November 2016

CONTENTS

- TD analysis is related to the assessment of transmitted E-field in the ground due to GPR dipole antenna by means of BEM.
- TD approach is based on the space-time integral equation of the Hallen type.
- The influence of the earth-air interface is taken into account via the simplified space-time reflection/transmission coefficient arising from the Modified Image Theory (MIT).

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EM Field Coupling to Overhead Wires

- The integral equation for the transient current along the **wire** is obtained by enforcing the condition for the tangential field at the wire surface:

$$E_z^{inc} + E_z^{sct} = 0$$

where E_z^{inc} is the incident electric field and the scattered electric field E_z^{inc} is expressed in terms of potentials:

$$\vec{E}^{sct}\Big|_{tan} = -\left(\frac{\partial \vec{A}}{\partial t} + \nabla \varphi\right)\Big|_{tan}$$

The vector and scalar potential, respectively, are given by:

$$\vec{A} = \frac{\mu}{4\pi} \iint_{S'} \frac{\vec{J}(\vec{r}', t-R/c)}{R} dS'$$

$$\varphi = \frac{1}{4\pi\varepsilon} \iint_{S'} \frac{\rho(\vec{r}', t-R/c)}{R} dS'$$

where charge and current densities, respectively, are related with the continuity equation:

$$\nabla \vec{J}_s = -\frac{\partial \rho_s}{\partial t}$$

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Combining previous equations leads to the integral equation for the transient current along the wire in free space:

$$-\varepsilon \frac{\partial E_z^{inc}}{\partial t} = \left[\frac{\partial^2}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right] \int_0^L \frac{I(z',t - R/c)}{4\pi R} dz'$$

Integrating the Pocklington equation yields the Hallen integral equation:

$$\int_0^L \frac{I(z',t - R/c)}{4\pi R} dz' = F_0(t - \frac{z}{c}) + F_L(t - \frac{L - z}{c}) + \frac{1}{2Z_0} \int_0^L E_{\check{z}}^{inc}(z',t - \frac{|z - z'|}{c}) dz'$$

- $I(z', t-R/c)$ is the unknown current to be determined,
- c is the velocity of light,
- Z_0 is the wave impedance of a free space
- $F_0(t)$; $F_L(t)$ are related with the reflections from wire ends

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Performing certain mathematical manipulations one obtains the Hallen integral equation for wire above a lossy half-space:

$$\int_0^L \frac{I(x',t-R/c)}{4\pi R}dx' - \int_{-\infty}^{t}\int_0^L r(\theta,\tau)\frac{I(x',t-R^*/c-\tau)}{4\pi R^*}dx'\,d\tau =$$

$$= F_0\left(t-\frac{x}{c}\right) + F_L\left(t-\frac{L-x}{c}\right) + \frac{1}{2Z_0}\int_0^L E_x^{exc}\left(x',t-\frac{|x-x'|}{c}\right)dx'$$

where the influence of the interface is taken into account via the space-time reflection coefficient:

$$r(\theta',t) = A\delta(t) + \frac{4\beta}{1-\beta^2}\frac{e^{-at}}{t}\sum_{m=1}^{\infty}(-1)^{m+1} m A^m I_m(\alpha t)$$

The time domain Hallen equation is solved via the Galerkin-Bubnov indirect boundary element approach.

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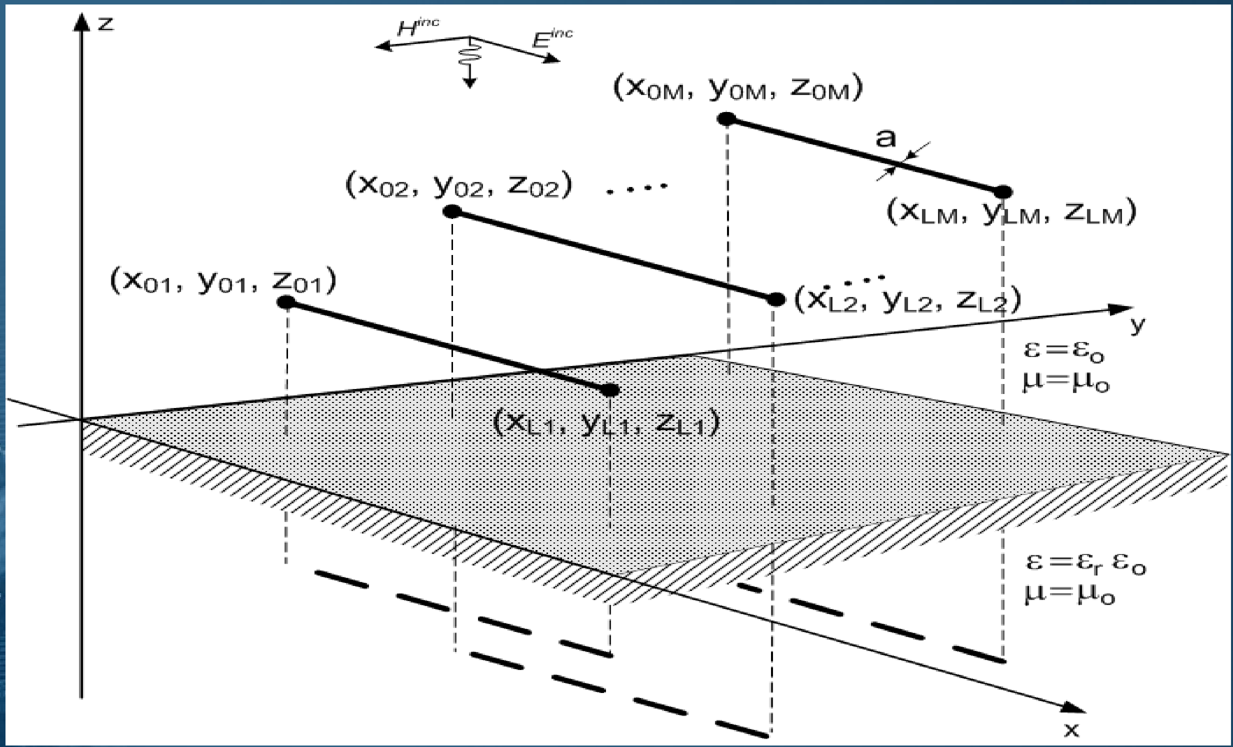
The antenna model: TD analysis of multiple wires

• The PEC wires of length *L* and radius *a*, at different heights *h* above a dielectric half-space, illuminated by an incident *E*-field, are of interest.



Straight thin wires at various heights above a dielectric half - space

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EM Field Coupling to Overhead Wires

Transient response of M parallel wires above a real ground is governed by the set of the coupled space-time Hallen integral equations:

$$\sum_{s=1}^{M}\int_{0}^{L_s}\frac{I_s(x',t-\frac{R_{vs}}{c})}{4\pi R_{vs}}dx'-\sum_{s=1}^{M}\int_{-\infty}^{t}\int_{0}^{L_s}r_{vs}(\theta,\tau)\frac{I_s(x',t-\frac{R^*_{vs}}{c}-\tau)}{4\pi R^*_{vs}}dx'd\tau$$
$$=F_{0v}(t-\frac{x-x_{0v}}{c})+F_{Lv}(t-\frac{x_{Lv}-x}{c})+\frac{1}{2Z_0}\int_{0}^{L_s}E^{exc}_{xv}(x',t-\frac{|x-x'|}{c})dx'$$

Unknown time signals $F_{0v}(t)$ and $F_L(t)$ related to the multiple reflections of transient currents at the wires free ends are given by:

$$F_{0v}(t)=\sum_{n=0}^{\infty}K_{0v}(t-\frac{2nL_v}{c})-\sum_{n=0}^{\infty}K_{Lv}(t-\frac{(2n+1)L_v}{c})$$

$$F_{Lv}(t)=\sum_{n=0}^{\infty}K_{Lv}(t-\frac{2nL_v}{c})-\sum_{n=0}^{\infty}K_{0v}(t-\frac{(2n+1)L_v}{c})$$

The auxilliary functions K are defined:

$$K_{0v}(t)=\sum_{s=1}^{M}\int_{0}^{L_s}\frac{I_s(x',t-\frac{R^{(0)}_{vs}}{c})}{4\pi R^{(0)}_{vs}}dx'-\sum_{s=1}^{M}\int_{-\infty}^{t}\int_{0}^{L_s}r_{vs}(\theta,\tau)\frac{I_s(x',t-\frac{R^{*(0)}_{vs}}{c}-\tau)}{4\pi R^{*(0)}_{vs}}dx'd\tau-$$
$$-\frac{1}{2Z_0}\int_{0}^{L_s}E^{exc}_{x}(x',t-\frac{|x-x'|}{c})dx'$$

$$K_{Lv}(t)=\sum_{s=1}^{M}\int_{0}^{L_s}\frac{I_s(x',t-\frac{R^{(L)}_{vs}}{c})}{4\pi R^{(L)}_{vs}}dx'-\sum_{s=1}^{M}\int_{-\infty}^{t}\int_{0}^{L_s}r_{vs}(\theta,\tau)\frac{I_s(x',t-\frac{R^{*(L)}_{vs}}{c}-\tau)}{4\pi R^{*(L)}_{vs}}dx'd\tau-$$
$$-\frac{1}{2Z_0}\int_{0}^{L_s}E^{exc}_{x}(x',t-\frac{|x-x'|}{c})dx'$$

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For the case of a dielectric half-space the set of Hallen integral equations simplifies into:

$$\sum_{s=1}^{M}\int_0^{L_s}\frac{I_s(x',t-\frac{R_{vs}}{c})}{4\pi R_{vs}}dx' - \sum_{s=1}^{M}\int_0^{L_s} r_{vs}(\theta)\frac{I_s(x',t-\frac{R^*_{vs}}{c})}{4\pi R^*_{vs}}dx' =$$

$$F_{0v}(t-\frac{x-x_{0v}}{c}) + F_{Lv}(t-\frac{x_{Lv}-x}{c}) + \frac{1}{2Z_0}\int_0^{L_s}E^{exc}_{xv}(x',\, t-\frac{|x-x'|}{c})dx'$$

Space dependent reflection coefficient is: $r_{vs}(\theta)=\frac{1-\beta}{1+\beta},\ \beta=\frac{\sqrt{\varepsilon_r-\sin^2\theta'}}{\varepsilon_r\cos\theta'},\qquad \theta'_{vs}=Arctg\frac{\sqrt{(x'-x)^2+(y'-y)^2_{vs}}}{z'+z}$

- For the normal incidence the total *E* - field is given by:

$$E^{exc}_{xv}(x',z,t)=E^{inc}_{xv}(x',t-T)+E^{ref}_{xv}(x',\, t-T)$$

T - the time required for the wave to travel from the highest wire to the height *z* of the observed *v-th* wire.

- The reflected field:

$$E^{ref}_{xv}(x',\, t-T)=r_{|\theta=0}\cdot E^{inc}_{xv}(x',t-T-\frac{2z}{c})$$

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Numerical solution

The set of Hallen integral equations is handled via the TD scheme of the **Galerkin-Bubnov Indirect Boundary Element Method** (GB-IBEM).

- local approximation for the current on a wire: $$I(x',t') = \{f\}^T \{I\}$$

- space-domain shape functions given by: $$f_r(x') = \frac{x_{r+1}-x'}{x_{r+1}-x_r} \qquad f_{r+1}(x') = \frac{x'-x_r}{x_{r+1}-x_r}$$

Applying the BEM discretisation leads to a local system of linear equations for the vth observed wire:

$$\sum_{s=1}^{M}\left[\int_{\Delta l_i}\int_{\Delta l_j}\frac{1}{4\pi R_{vs}}\{f\}_j\{f\}_i^T\,dx'dx\{I_s\}\Big|_{t-\frac{R_{vs}}{c}} - \int_{\Delta l_i}\int_{\Delta l_j}\frac{r_{vs}(\theta)}{4\pi R_{vs}^*}\{f\}_j\{f\}_i^T\,dx'dx\{I_s\}\Big|_{t-\frac{R_{vs}^*}{c}}\right]$$
$$=\frac{1}{2Z_0}\int_{\Delta l_i}\int_{\Delta l_j}E_{xv}^{exc}\left(x',t-\frac{|x-x'|}{c}\right)\{f\}_j\,dx'dx$$
$$+\int_{\Delta l_j}F_0\left(t-\frac{x-x_{0v}}{c}\right)\{f\}_j\,dx+\int_{\Delta l_j}F_L\left(t-\frac{x_{Lv}-x}{c}\right)\{f\}_j\,dx$$

i,j=1,2..N - index of the elements (*s*-th source and the *v*-th observed wire, respectively *N* - total number of segments, *M* - actual number of wires.

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the matrix equation:

$$\sum_{s=1}^{M}[A_{vs}]\{I_s\}\Big|_{t-\frac{R_{vs}}{c}} - \sum_{s=1}^{M}\left[A^*_{vs}\right]\{I_s\}\Big|_{t-\frac{R^*_{vs}}{c}} =$$

$$= [B_v]\{E_v\}\Big|_{t-\frac{|x-x'|}{c}} + \sum_{s=1}^{M}[C_{vs}]\left\{\sum_{n=0}^{\infty} I_s^n\right\}\Bigg|_{t-\frac{x-x_{0v}}{c}-\frac{2n}{c}L_v-\frac{R_{vs}^{(0)}}{c}} - \sum_{s=1}^{M}\left[C^*_{vs}\right]\left\{\sum_{n=0}^{\infty} I_s^n\right\}\Bigg|_{t-\frac{x-x_{0v}}{c}-\frac{2n}{c}L_v-\frac{R_{vs}^{*(0)}}{c}} - [D_v]\left\{\sum_{n=0}^{\infty} E_v^n\right\}\Bigg|_{t-\frac{x-x_{0v}}{c}-\frac{2n}{c}L_v-\frac{|x'-x_{0v}|}{c}} -$$

$$- \sum_{s=1}^{M}[E_{vs}]\left\{\sum_{n=0}^{\infty} I_s^n\right\}\Bigg|_{t-\frac{x-x_{0v}}{c}-\frac{2n+1}{c}L_v-\frac{R_{vs}^{(L)}}{c}} + \sum_{s=1}^{M}\left[E^*_{vs}\right]\left\{\sum_{n=0}^{\infty} I_s^n\right\}\Bigg|_{t-\frac{x-x_{0v}}{c}-\frac{2n+1}{c}L_v-\frac{R_{vs}^{*(L)}}{c}} + [D_v]\left\{\sum_{n=0}^{\infty} E_v^n\right\}\Bigg|_{t-\frac{x-x_{0v}}{c}-\frac{2n+1}{c}L_v-\frac{|x_{Lv}-x'|}{c}} + \sum_{s=1}^{M}[E_{vs}]\left\{\sum_{n=0}^{\infty} I_s^n\right\}\Bigg|_{t-\frac{x_{Lv}-x}{c}-\frac{2n}{c}L_v-\frac{R_{vs}^{(L)}}{c}} -$$

$$- \sum_{s=1}^{M}\left[E^*_{vs}\right]\left\{\sum_{n=0}^{\infty} I_s^n\right\}\Bigg|_{t-\frac{x_{Lv}-x}{c}-\frac{2n}{c}L_v-\frac{R_{vs}^{*(L)}}{c}} - [D_v]\left\{\sum_{n=0}^{\infty} E_v^n\right\}\Bigg|_{t-\frac{x_{Lv}-x}{c}-\frac{2n}{c}L_v-\frac{|x_{Lv}-x'|}{c}} - \sum_{s=1}^{M}[C_{vs}]\left\{\sum_{n=0}^{\infty} I_s^n\right\}\Bigg|_{t-\frac{x_{Lv}-x}{c}-\frac{2n+1}{c}L_v-\frac{R_{vs}^{(0)}}{c}} + \sum_{s=1}^{M}\left[C^*_{os}\right]\left\{\sum_{n=0}^{\infty} I_s^n\right\}\Bigg|_{t-\frac{x_{Lo}-x}{c}-\frac{2n+1}{c}L_o-\frac{R_{os}^{*(0)}}{c}} +$$

$$+ [D_o]\left\{\sum_{n=0}^{\infty} E_o^n\right\}\Bigg|_{t-\frac{x_{Lo}-x}{c}-\frac{2n+1}{c}L_o-\frac{|x'-x_{0o}|}{c}}$$

the space dependent matrices:

$$[A_{vs}] = \int_{\Delta l_j}\int_{\Delta l_i} \frac{1}{4\pi R_{vs}}\{f\}_j\{f\}_i^T\, dx'dx\,; \quad \left[A^*_{vs}\right] = \int_{\Delta l_j}\int_{\Delta l_i} \frac{r_{vs}(\theta)}{4\pi R^*_{vs}}\{f\}_j\{f\}_i^T\, dx'dx$$

$$[B_v] = \frac{1}{2Z_0}\int_{\Delta l_j}\int_{\Delta l_i} \{f\}_j\{f\}_i^T\, dx'dx \quad ,[C_{vs}] = \int_{\Delta l_j}\int_{\Delta l_i} \frac{1}{4\pi R_{vs}^{(0)}}\{f\}_j\{f\}_i^T\, dx'dx$$

$$\left[C^*_{vs}\right] = \int_{\Delta l_j}\int_{\Delta l_i} \frac{r_{vs}(\theta)}{4\pi R_{vs}^{*(0)}}\{f\}_j\{f\}_i^T\, dx'dx \quad [D_v] = \frac{1}{2Z_0}\int_{\Delta l_j}\int_{\Delta l_i} \{f\}_j\{f\}_i^T\, dx'dx$$

$$[E_{vs}] = \int_{\Delta l_j}\int_{\Delta l_i} \frac{1}{4\pi R_{vs}^{(L)}}\{f\}_j\{f\}_i^T\, dx'dx \quad ,\left[E^*_{vs}\right] = \int_{\Delta l_j}\int_{\Delta l_i} \frac{r_{vs}(\theta)}{4\pi R_{vs}^{*(L)}}\{f\}_j\{f\}_i^T\, dx'dx$$

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EM Field Coupling to Overhead Wires

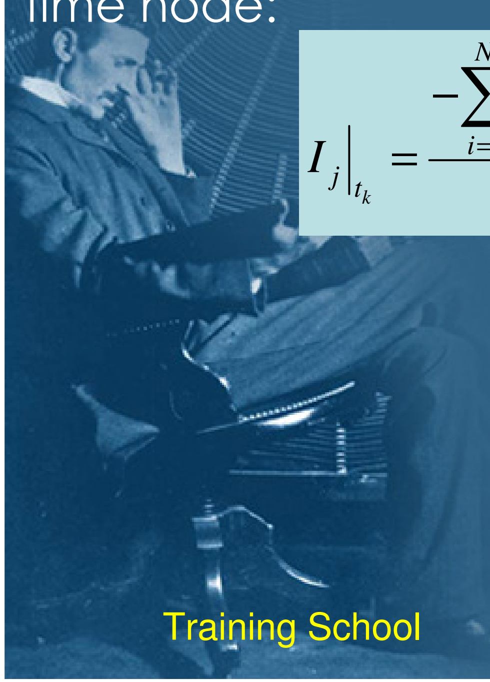
TD antenna model

- The weighted residual approach in the time domain:

$$\int_{t_k}^{t_k+\Delta t}\left(\left[A\right]\{I\}\Big|_{t-\frac{R_{vs}}{c}}-\left[A^*\right]\{I\}\Big|_{t-\frac{R_{vs}^*}{c}}-\{g\}\theta_k\right)dt=0;\ k=1,2,\ \ldots,\ N_t$$

- the recurrence formula for the transient current at *j*-th space node and *k*-th time node:

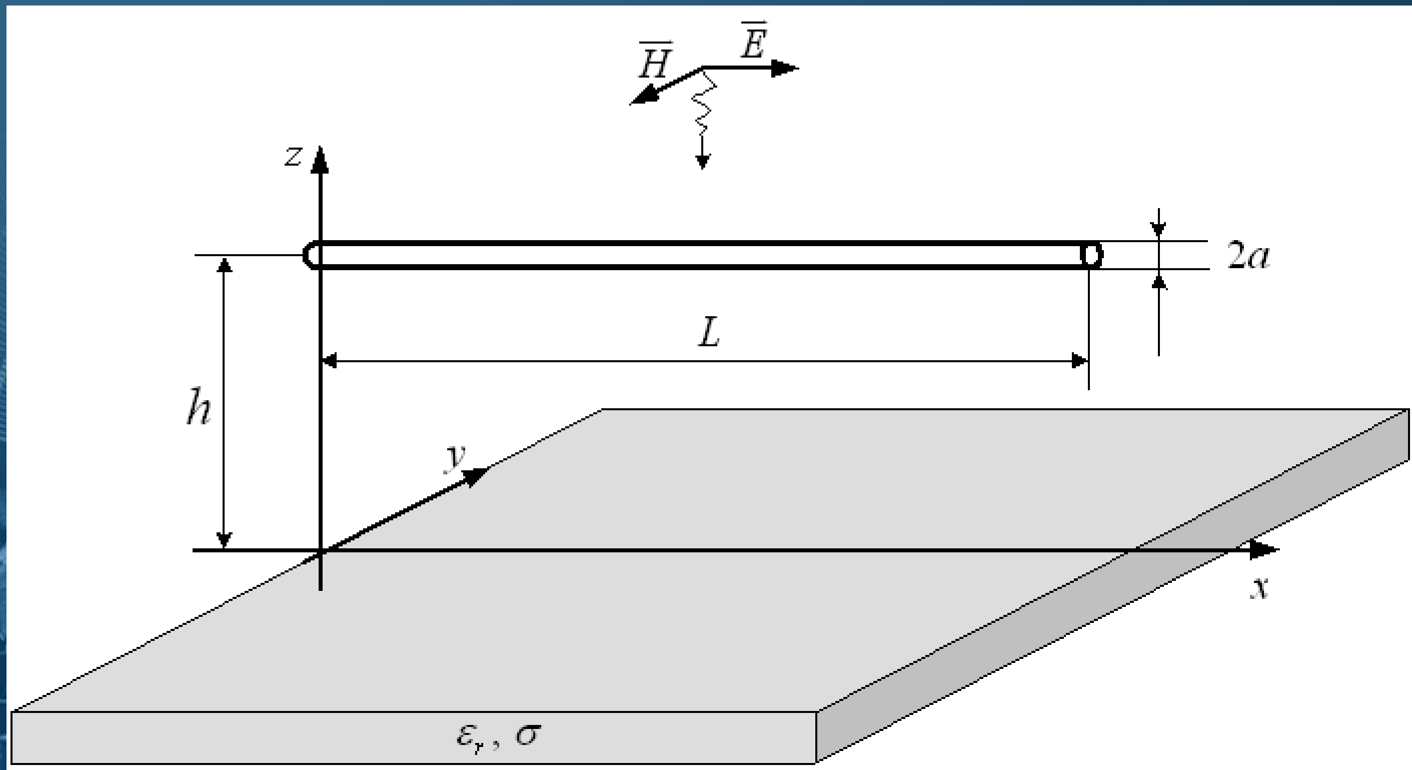
$$I_j\Big|_{t_k}=\frac{-\sum_{i=1}^{N}\left(\overline{A_{ji}}\,I_i\Big|_{t_k-\frac{R_{vs}}{c}}+A_{ji}^*\,I_i\Big|_{t_k-\frac{R_{vs}^*}{c}}\right)+g_j\Big|_{all\ previous\ discrete\ instants}}{A_{jj}}$$



EM Field Coupling to Overhead Wires

TD antenna model: Inclusion of ground conductivity

Geometry observed – scattering on a thin wire above lossy ground:



EM Field Coupling to Overhead Wires

TD antenna model: Inclusion of ground conductivity

- The space-time reflection coefficient can be written, as follows:

$$r(\theta,\tau) = r'(\theta,\tau) + r''(\theta,\tau)$$

where:

$$r'(\theta,t) = K\delta(t)$$

$$r''(\theta,t) = \frac{4\beta}{1-\beta^2}\frac{e^{-\alpha t}}{t}\sum_{n=1}^{\infty}(-1)^{n+1} nK^n I_n(\alpha t)$$

$$K = \frac{1-\beta}{1+\beta}$$

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TD antenna model: Inclusion of ground conductivity

- Inclusion of $r''(\theta, t)$ into the model leads to the following matrix equation:

$$[A]\{I\}_i\Big|_{t-\frac{R}{c}} - \left[A^*\right]\{I\}_i\Big|_{t-\frac{R^*}{c}} - \{\hat{A}\}\Big|_{t-\frac{R^*}{c}} = [B]\{E\}\Big|_{t-\frac{|x-x'|}{c}}$$

$$+[C]\left\{\sum_{n=0}^{\infty} I^n\right\}_i\Bigg|_{t-\frac{R_0}{c}-\frac{2nL}{c}-\frac{x}{c}} - \left[C^*\right]\left\{\sum_{n=0}^{\infty} I^n\right\}_i\Bigg|_{t-\frac{R_0^*}{c}-\frac{2nL}{c}-\frac{x}{c}} - \left\{\sum_{n=0}^{\infty} \hat{C}^n\right\}\Bigg|_{t-\frac{R_0^*}{c}-\frac{2nL}{c}-\frac{x}{c}}$$

$$-[B]\left\{\sum_{n=0}^{\infty} E^n\right\}\Bigg|_{t-\frac{x'}{c}-\frac{2nL}{c}-\frac{x}{c}} - [D]\left\{\sum_{n=0}^{\infty} I^n\right\}_i\Bigg|_{t-\frac{R_L}{c}-\frac{(2n+1)L}{c}-\frac{x}{c}}$$

$$+\left[D^*\right]\left\{\sum_{n=0}^{\infty} I^n\right\}_i\Bigg|_{t-\frac{R_L^*}{c}-\frac{(2n+1)L}{c}-\frac{x}{c}} + [B]\left\{\sum_{n=0}^{\infty} E^n\right\}\Bigg|_{t-\frac{L-x'}{c}-\frac{(2n+1)L}{c}-\frac{x}{c}} + \left\{\sum_{n=0}^{\infty} \hat{D}^n\right\}\Bigg|_{t-\frac{R_L^*}{c}-\frac{(2n+1)L}{c}-\frac{x}{c}}$$

$$+[D]\left\{\sum_{n=0}^{\infty} I^n\right\}_i\Bigg|_{t-\frac{R_L}{c}-\frac{2nL}{c}-\frac{L-x}{c}} - \left[D^*\right]\left\{\sum_{n=0}^{\infty} I^n\right\}_i\Bigg|_{t-\frac{R_L^*}{c}-\frac{2nL}{c}-\frac{L-x}{c}} + \left\{\sum_{n=0}^{\infty} \hat{D}^n\right\}\Bigg|_{t-\frac{R_L^*}{c}-\frac{2nL}{c}-\frac{L-x}{c}}$$

$$-[B]\left\{\sum_{n=0}^{\infty} E^n\right\}\Bigg|_{t-\frac{L-x'}{c}-\frac{2nL}{c}-\frac{L-x}{c}} - [C]\left\{\sum_{n=0}^{\infty} I^n\right\}_i\Bigg|_{t-\frac{R_0}{c}-\frac{(2n+1)L}{c}-\frac{L-x}{c}}$$

$$+\left[C^*\right]\left\{\sum_{n=0}^{\infty} I^n\right\}_i\Bigg|_{t-\frac{R_0^*}{c}-\frac{(2n+1)L}{c}-\frac{L-x}{c}} + [B]\left\{\sum_{n=0}^{\infty} E^n\right\}\Bigg|_{t-\frac{x'}{c}-\frac{(2n+1)L}{c}-\frac{L-x}{c}} - \left\{\sum_{n=0}^{\infty} \hat{C}^n\right\}\Bigg|_{t-\frac{R_0^*}{c}-\frac{(2n+1)L}{c}-\frac{L-x}{c}}$$

EM Field Coupling to Overhead Wires

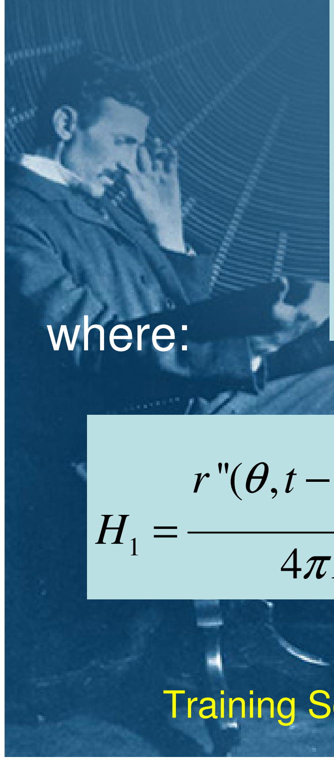
TD antenna model: Inclusion of ground conductivity

- Additional vectors are expressed as follows:

$$\left\{\hat{A}\right\} = \int_{0}^{t-\frac{R^*}{c}} \int_{\Delta l_j} \int_{\Delta l_i} \{f\}_j \{f\}_i^T H_1 dx' dx \{I(\tau)\}_i d\tau$$

$$\left\{\hat{C}^n\right\} = \int_{0}^{t-\frac{R_0^*}{c}-\frac{2nL}{c}-\frac{x}{c}} \int_{\Delta l_j} \int_{\Delta l_i} \{f\}_j \{f\}_i^T H_2 dx' dx \{I(\tau)\}_i d\tau$$

$$\left\{\hat{D}^n\right\} = \int_{0}^{t-\frac{R_L^*}{c}-\frac{(2n+1)L}{c}-\frac{x}{c}} \int_{\Delta l_j} \int_{\Delta l_i} \{f\}_j \{f\}_i^T H_3 dx' dx \{I(\tau)\}_i d\tau$$



where:

$$H_1 = \frac{r''(\theta, t-\frac{R^*}{c}-\tau)}{4\pi R^*}; \qquad H_2 = \frac{r''(\theta, t-\frac{R_0^*}{c}-\frac{2nL}{c}-\frac{x}{c}-\tau)}{4\pi R_0^*}; \qquad H_3 = \frac{r''(\theta, t-\frac{R_L^*}{c}-\frac{(2n+1)L}{c}-\frac{x}{c}-\tau)}{4\pi R_0^*}$$

EM Field Coupling to Overhead Wires

TD antenna model: Inclusion of ground conductivity

- Assembly into global matrix system yields:

$$\left[A\right]\left\{I\right\}\Big|_{t-\frac{R}{c}}=\left\{g\right\}\Big|_{\substack{\text{previous time}\\ \text{instants}}}+\left\{\hat{g}\right\}\Big|_{\substack{\text{previous time}\\ \text{instants}}}$$

where:

$$\left\{\hat{g}\right\}=\left\{\hat{A}\right\}\Big|_{t-\frac{R^*}{c}}-\left\{\sum_{n=0}^{\infty}\hat{C}^n\right\}\Bigg|_{t-\frac{R_0^*}{c}-\frac{2nL}{c}-\frac{x}{c}}+\left\{\sum_{n=0}^{\infty}\hat{D}^n\right\}\Bigg|_{t-\frac{R_L^*}{c}-\frac{(2n+1)L}{c}-\frac{x}{c}}+\left\{\sum_{n=0}^{\infty}\hat{D}^n\right\}\Bigg|_{t-\frac{R_L^*}{c}-\frac{2nL}{c}-\frac{L-x}{c}}-\left\{\sum_{n=0}^{\infty}\hat{C}^n\right\}\Bigg|_{t-\frac{R_0^*}{c}-\frac{(2n+1)L}{c}-\frac{L-x}{c}}$$

$$\begin{aligned}\left\{g\right\}=\left[A^*\right]\left\{I\right\}\Big|_{t-\frac{R^*}{c}}+\left[B\right]\left\{E\right\}\Big|_{t-\frac{|x-x'|}{c}}\\
+\left[C\right]\left\{\sum_{n=0}^{\infty}I^n\right\}\Bigg|_{t-\frac{R_0}{c}-\frac{2nL}{c}-\frac{x}{c}}-\left[C^*\right]\left\{\sum_{n=0}^{\infty}I^n\right\}\Bigg|_{t-\frac{R_0^*}{c}-\frac{2nL}{c}-\frac{x}{c}}\\
-\left[B\right]\left\{\sum_{n=0}^{\infty}E^n\right\}\Bigg|_{t-\frac{x'}{c}-\frac{2nL}{c}-\frac{x}{c}}-\left[D\right]\left\{\sum_{n=0}^{\infty}I^n\right\}\Bigg|_{t-\frac{R_L}{c}-\frac{(2n+1)L}{c}-\frac{x}{c}}\\
+\left[D^*\right]\left\{\sum_{n=0}^{\infty}I^n\right\}\Bigg|_{t-\frac{R_L^*}{c}-\frac{(2n+1)L}{c}-\frac{x}{c}}+\left[B\right]\left\{\sum_{n=0}^{\infty}E^n\right\}\Bigg|_{t-\frac{L-x'}{c}-\frac{(2n+1)L}{c}-\frac{x}{c}}\\
+\left[D\right]\left\{\sum_{n=0}^{\infty}I^n\right\}\Bigg|_{t-\frac{R_L}{c}-\frac{2nL}{c}-\frac{L-x}{c}}-\left[D^*\right]\left\{\sum_{n=0}^{\infty}I^n\right\}\Bigg|_{t-\frac{R_L^*}{c}-\frac{2nL}{c}-\frac{L-x}{c}}\\
-\left[B\right]\left\{\sum_{n=0}^{\infty}E^n\right\}\Bigg|_{t-\frac{L-x'}{c}-\frac{2nL}{c}-\frac{L-x}{c}}-\left[C\right]\left\{\sum_{n=0}^{\infty}I^n\right\}\Bigg|_{t-\frac{R_0}{c}-\frac{(2n+1)L}{c}-\frac{L-x}{c}}\\
+\left[C^*\right]\left\{\sum_{n=0}^{\infty}I^n\right\}\Bigg|_{t-\frac{R_0^*}{c}-\frac{(2n+1)L}{c}-\frac{L-x}{c}}+\left[B\right]\left\{\sum_{n=0}^{\infty}E^n\right\}\Bigg|_{t-\frac{x'}{c}-\frac{(2n+1)L}{c}-\frac{L-x}{c}}\end{aligned}$$

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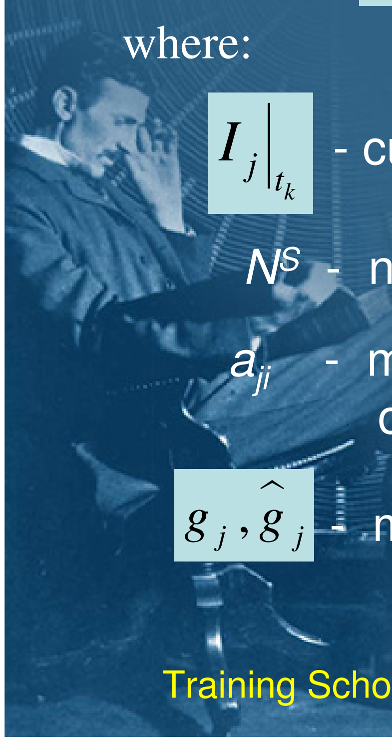
EM Field Coupling to Overhead Wires

TD antenna model: Inclusion of ground conductivity

- After time sampling, recurrent formula for the unknown current:

$$I_j\Big|_{t_k} = \frac{\sum_{i=1}^{N^s} a_{ji}\, I_j\Big|_{t_k - \frac{R}{c}} - g_j\Big|_{\substack{previous\ time\\ instants}} - \hat{g}_j\Big|_{\substack{previous\ time\\ instants}}}{a_{jj}}$$

where:



$I_j\Big|_{t_k}$ - current for the *j*-th space node and *k*-th time node

N^S - number of space elements

a_{ji} - member of matrix *[A]* for *i*-th source space node and *j*-th observation space node, where $i \neq j$

$g_j, \hat{g}_j$ - member of vectors $\{g\}, \{\hat{g}\}$ for *j j*-th observation space node

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Time domain analysis: Formulation

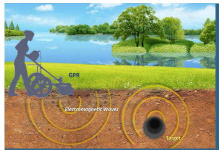
- The far field formula for the field in the air, reflected from the ground:

$$E_x(r,t) = -\frac{\mu_0}{4\pi}\left(\int_0^L \frac{\partial}{\partial t}\frac{I(x',t')}{R_1}dx' - \int_{-\infty}^{t} r(\theta,\tau)\int_0^L \frac{\partial}{\partial t}\frac{I(x',t_1'-\tau)}{R_1^*}dx'd\tau\right)$$

- The space-time reflection coefficient (MIT):

$$r_{ref}^{MIT} = -\frac{\tau_1}{\tau_2}\left[\delta(t) + \frac{1}{\tau_2}\left(1-\frac{\tau_1}{\tau_2}\right)e^{-t/\tau_2}\right]$$

$$\tau_1 = \frac{\varepsilon_r - 1}{\sigma}\varepsilon_0, \quad \tau_2 = \frac{\varepsilon_r + 1}{\sigma}\varepsilon_0$$



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Time domain analysis: Formulation

- the space-time far field formula for the field transmitted into the ground:

$$E_x^{tr}(r,t)=\frac{\mu_0}{4\pi}\int_{-\infty}^{t}\int_{0}^{L}\Gamma_{tr}^{MIT}(\theta,\tau)\frac{\partial I(x',t-R''/v-\tau)}{\partial t}\frac{e^{-\frac{1}{\tau_g}\frac{R''}{v}}}{R''}dx'\,d\tau$$

- the space-time transmission coefficient (MIT):

$$\Gamma_{tr}^{MIT}=\frac{2\tau_1}{\tau_2}\left[\delta(t)+\frac{1}{\tau_2}\left(1-\frac{\tau_2}{\tau_1}\right)e^{-t/\tau_2}\right]$$

$$\tau_1=\frac{\varepsilon_r}{\sigma}\varepsilon_0,\quad \tau_2=\frac{\varepsilon_r+1}{\sigma}\varepsilon_0,\quad t''=t-\frac{R''}{v}-\tau$$

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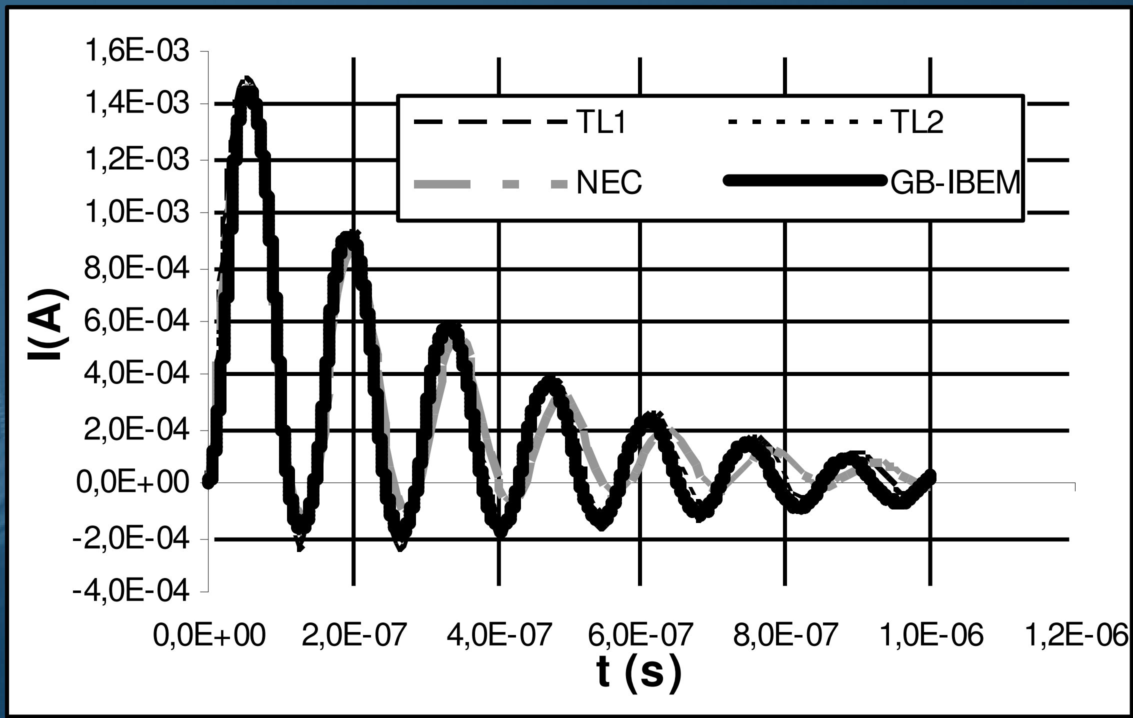
EM Field Coupling to Overhead Wires

Computational example: Transient response at the center of the line

The excitation: EMP
normal incidence

E_0=1.1V/m, $a=7.92*10^4 s^{-1}$, $b=4*10^4 s^{-1}$

$$E_x^{inc} = E_0\left(e^{-at} - e^{-bt}\right)$$



Transient current at the center of the line above dielectric half-space (L=20m,h=1m, ε_r=10)

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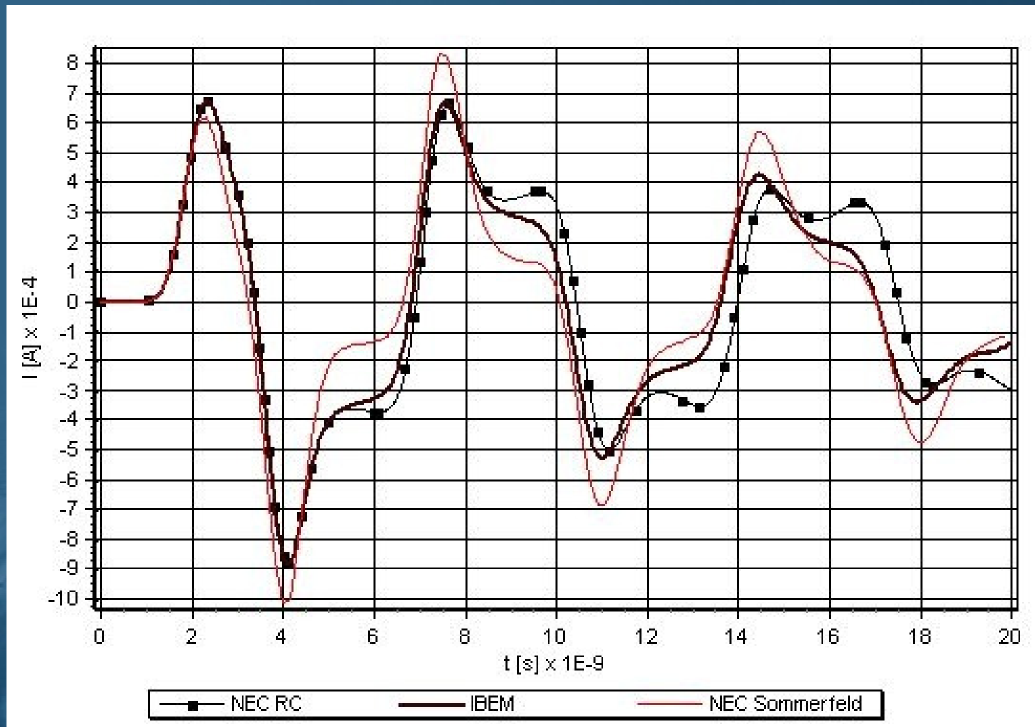
EM Field Coupling to Overhead Wires

Computational example: Transient response at the center of the line

The excitation: Gaussian pulse

$$E_x^{inc} = E_0 e^{-g^2 (t-t_0)^2}$$



Transient current induced at the center of the wire above a lossy ground
(L=1m, ε_r=10, σ=10mS/m)

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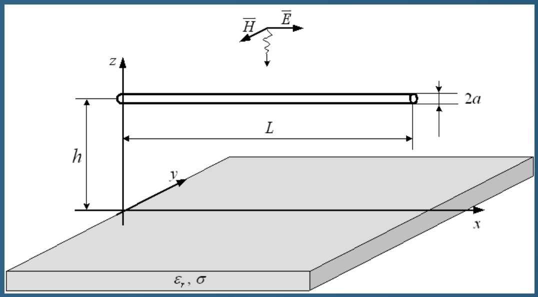
EM Field Coupling to Overhead Wires

Computational example: Transient response at the center of the line

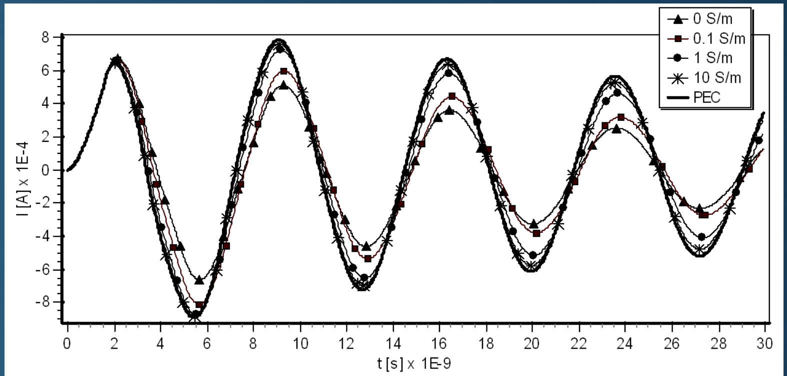
Configuration:



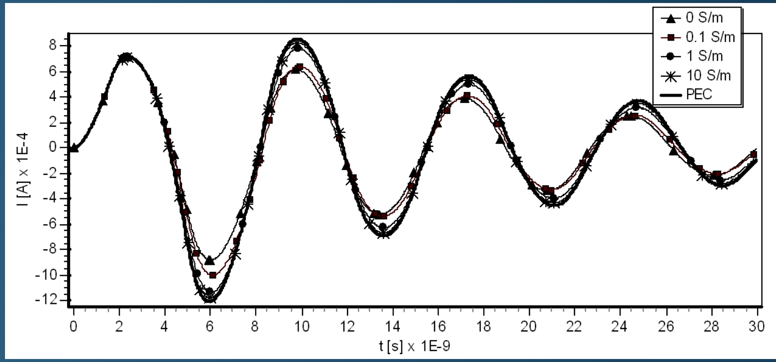
- *L*=1 m
- *a*=2 mm
- ε_r=10
- Incident electric field:

$$E_x^{inc}(t) = E_0(e^{-at} - e^{-bt})$$

E_0=1 V/m
a=4*10^7 s^{-1}
b=6*10^8 s^{-1}



h=0.25 m



h=0.5 m

Transient current at the wire centerfor various conductivities

EM Field Coupling to Overhead Wires

TD antenna model: Inclusion of ground conductivity

- Notes

- GB-IBEM expanded to numerically model ground conductivity
- Time dependent part of the reflection coefficient is modeled via additional vectors
- Convolutions integrals highly computationally inefficient
- Further modifications regarding computational efficiency necessary

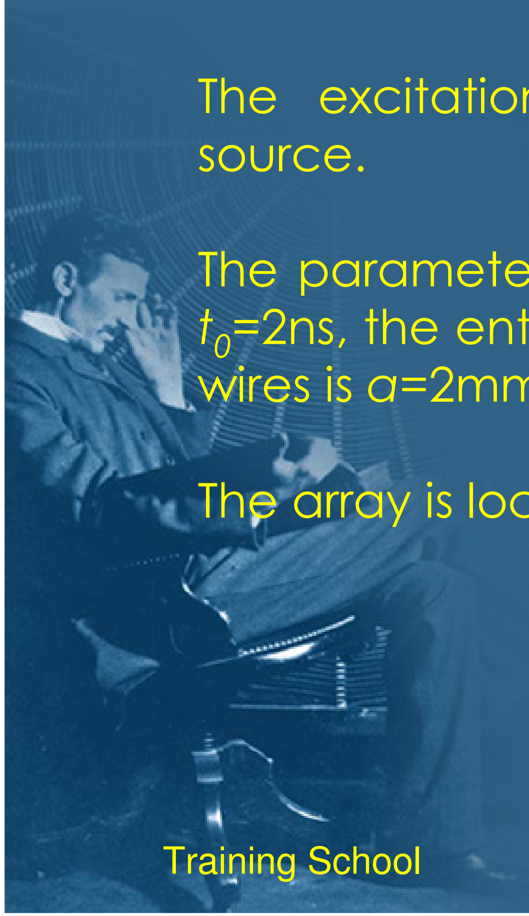
•Computational examples

- two-wire array

The excitation is a time-dependent Gaussian pulse voltage source.

The parameters of the Gaussian pulse are: V_0=1V, g=2*$10^9 s^{-1}$ and t_0=2ns, the entire length of the wires is L=1 m, while the radius of all wires is a=2mm.

The array is located above half-space (ε_r =10).



•Computational examples

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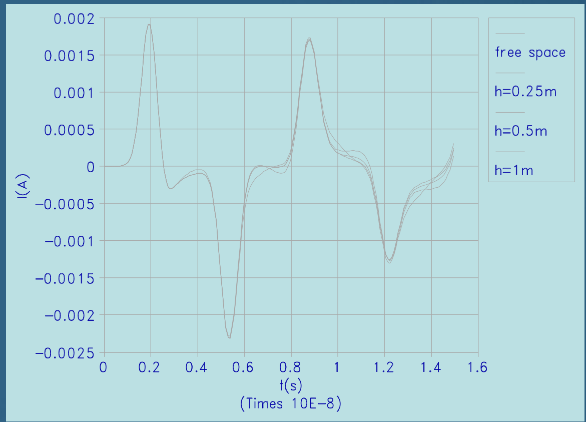
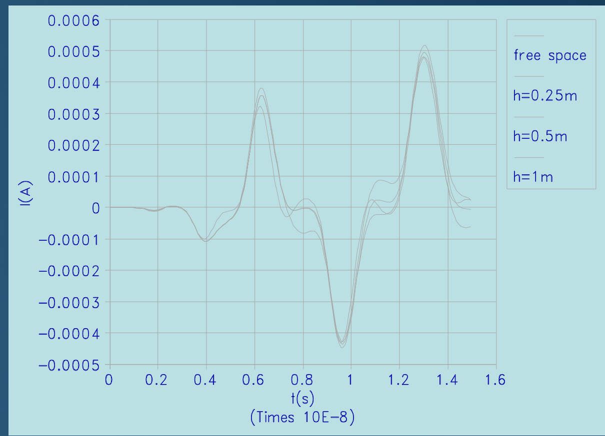
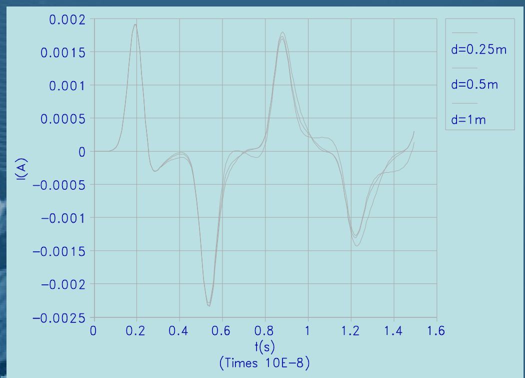


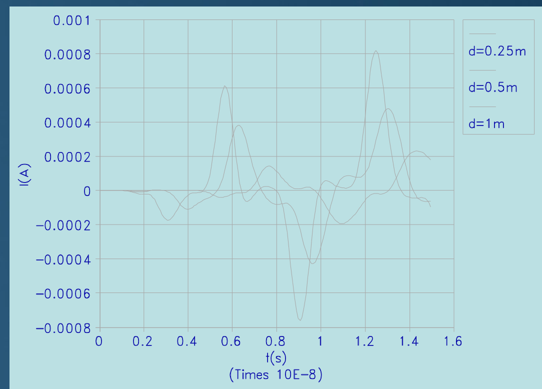
Current at the centre of the wire *A* (active wire) versus time with height *h* over interface as parameter (*L*=1m, *a*=2mm, ε_r=9, *h*=0.25m)



Current at the centre of wired *B* (passive wire) versus time with height *h* over interface as parameter (*L*=1m, *a*=2mm, ε_r=9, *h*=0.25m).



Current at the centre of wire A (active wire) versus time with distance *d* between the wires as a parameter (*L*=1m, *a*=2mm, ε_r=9, *d*=0.5m)



Current at the centre of wire *B* (passive wire) versus time with distance *d* between the wires as a parameter (*L*=1m, *a*=2mm, ε_r=9, *d*=0.5m)

•Computational examples

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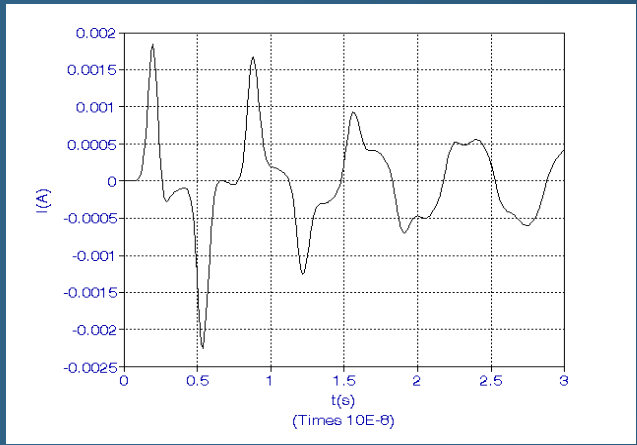
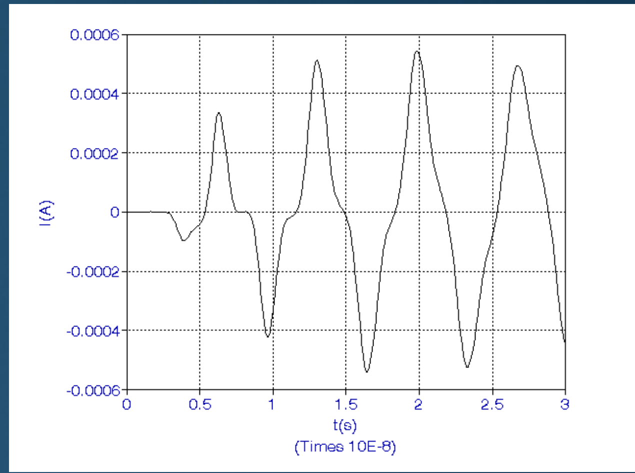
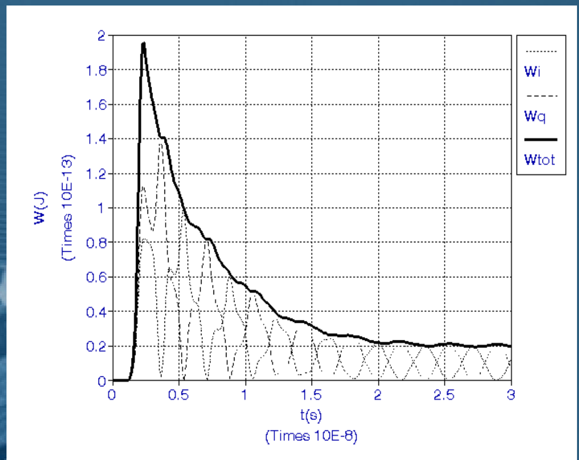


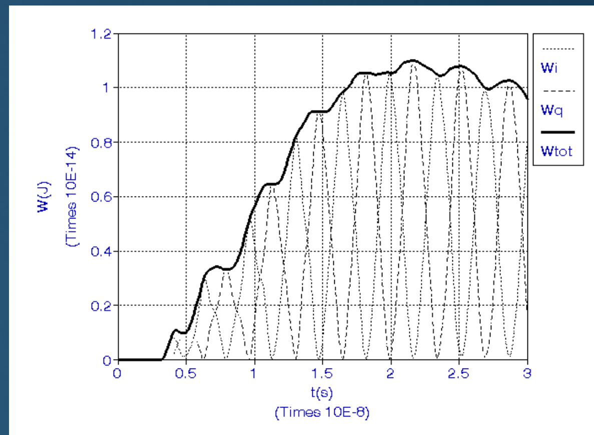
Transient current induced at the center of the active wire



Transient current induced at the center of the passive wires



The H-field (W_I) E-field (W_q) and total energy (W_{tot}) energy measures as afunction of time for the active wire



The H-field (W_I) E-field (W_q) and total energy (W_{tot}) energy measures as a function of time for the passive wire

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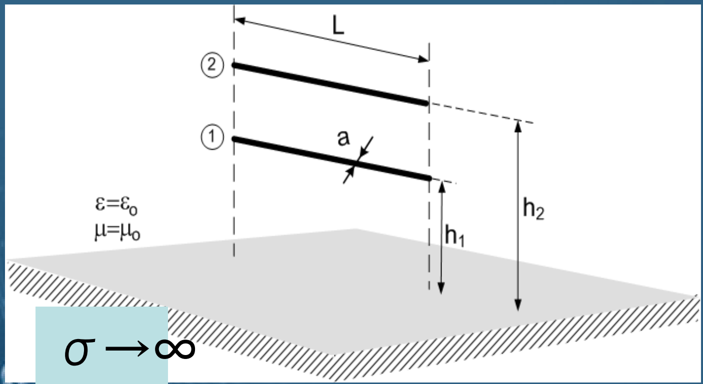
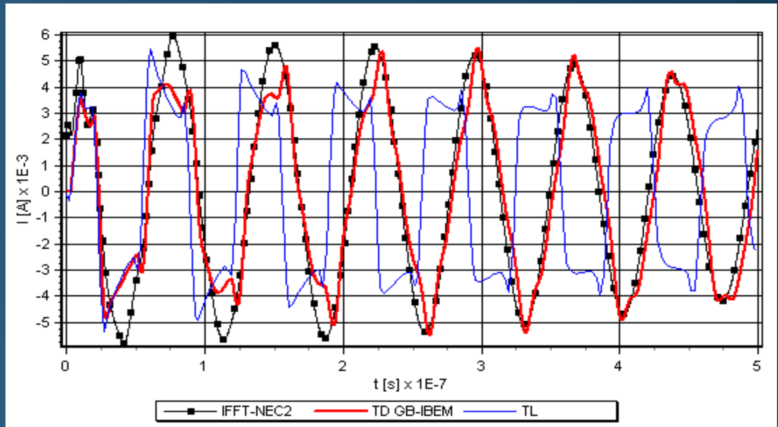
EM Field Coupling to Overhead Wires

Computational example: *Transient response of a two -wire array above a PEC ground - comparison between IFFT-NEC2, GB-IBEM and TL*

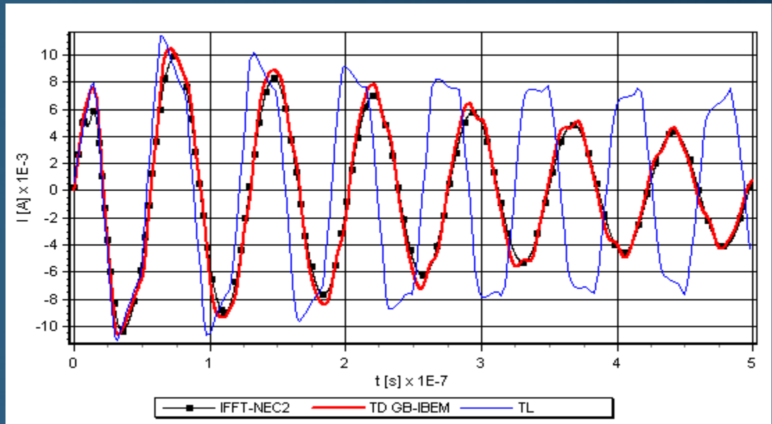


L=10m, a=2cm
h_1=1m, h_2=2m

Excitation: EMP
E_0=1V/m, $a=4*10^7 s^{-1}$, $b=6*10^8 s^{-1}$



Transient current induced at the center of wire 1



Transient current induced at the center of wire 2

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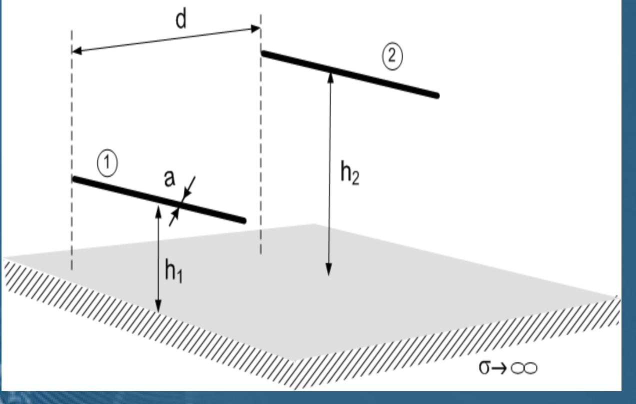
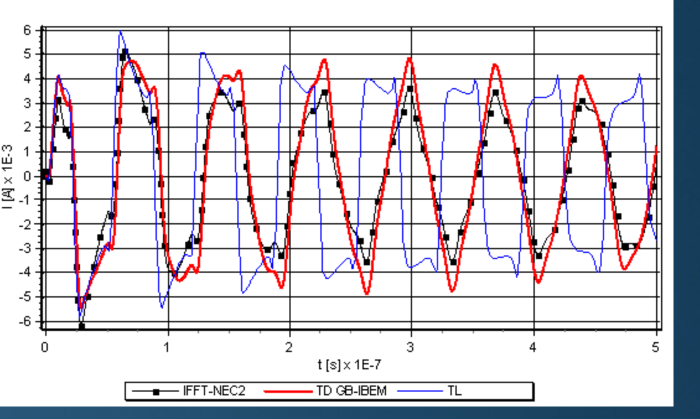
EM Field Coupling to Overhead Wires

Computational example: *Transient response of a two -wire array above a PEC ground - comparison between IFFT-NEC2, GB-IBEM and TL*

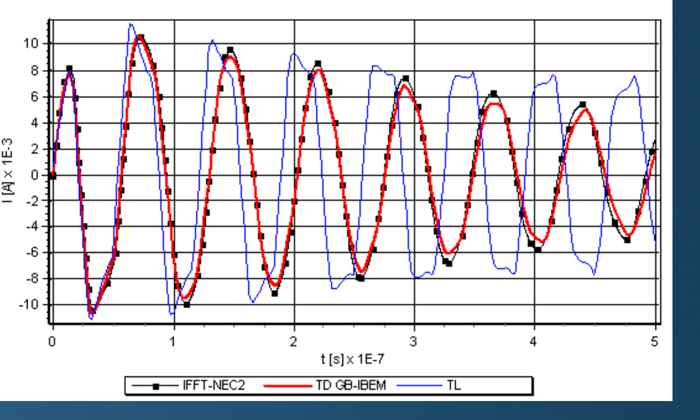




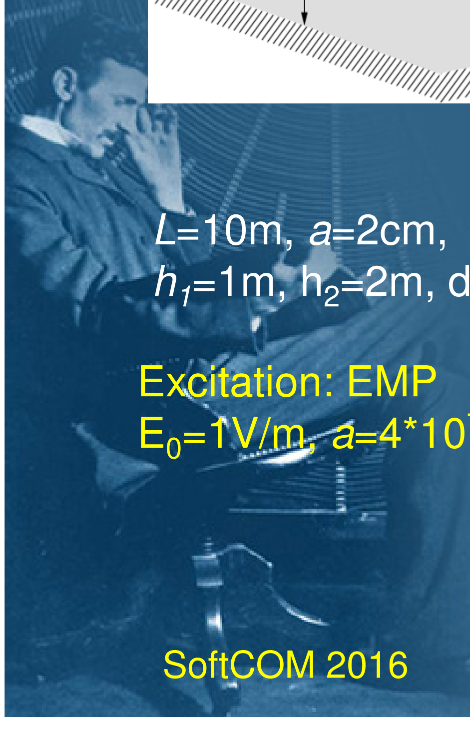
Transient current induced at the center of wire 1

L=10m, a=2cm,
h_1=1m, h_2=2m, d=1m

Excitation: EMP
E_0=1V/m, a=4*10^{7}s^{-1} , b=6*10^{8}s^{-1}



Transient current induced at the center of wire 2



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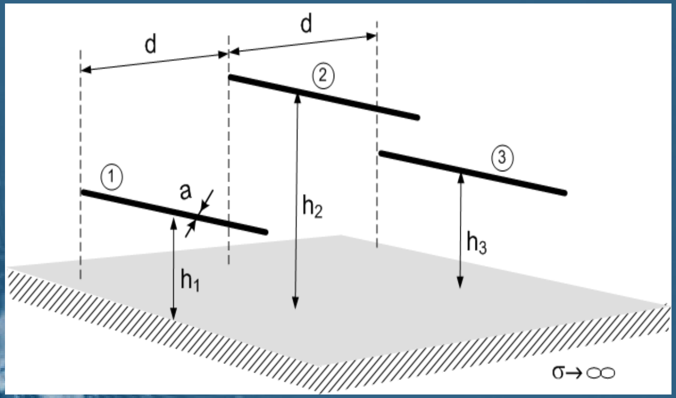
EM Field Coupling to Overhead Wires

Computational example: *Transient response of a three -wire array above a PEC ground - comparison between IFFT-NEC2, GB-IBEM and TL*

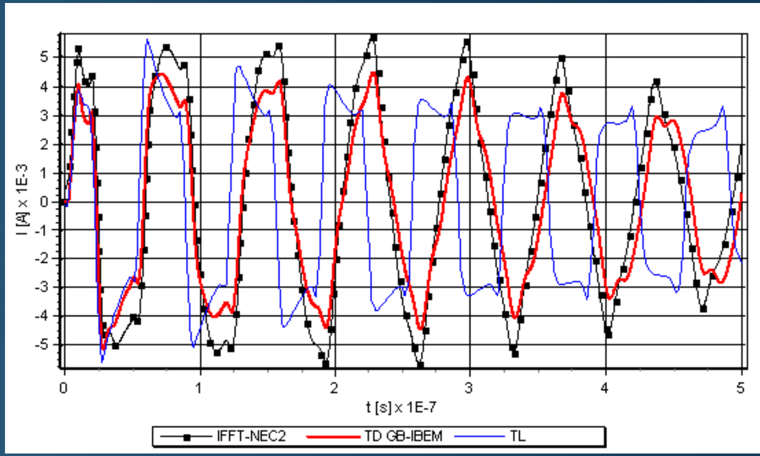


L=10m, a=2cm,

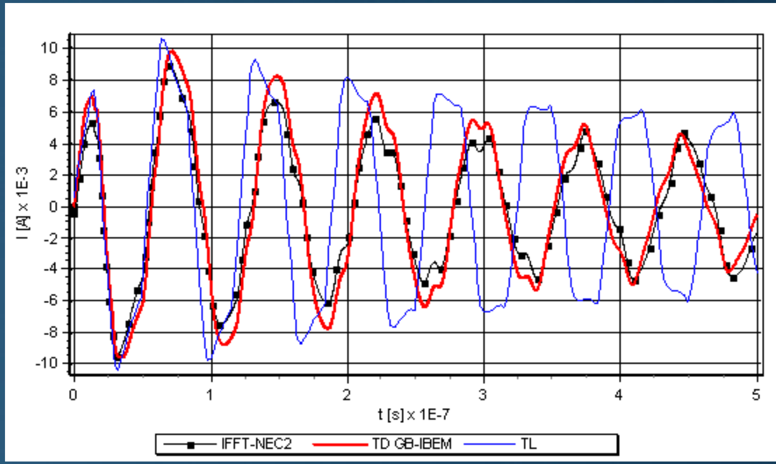
$h_1=h_3$=1m, h_2=2m, d=1m

Excitation: EMP

E_0=1V/m, $a=4*10^7 s^{-1}$, $b=6*10^8 s^{-1}$



Transient current induced at the center of wires 1 and 3



Transient current induced at the center of wire 2

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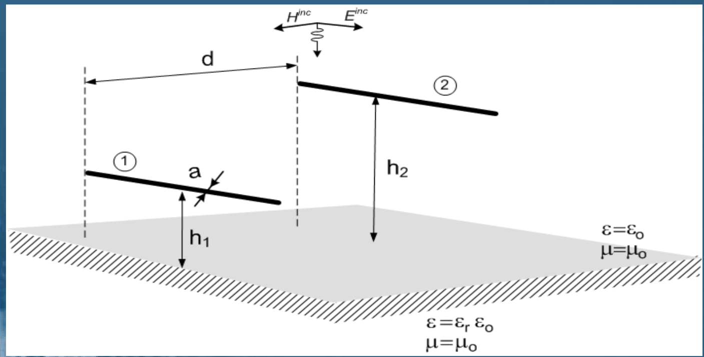
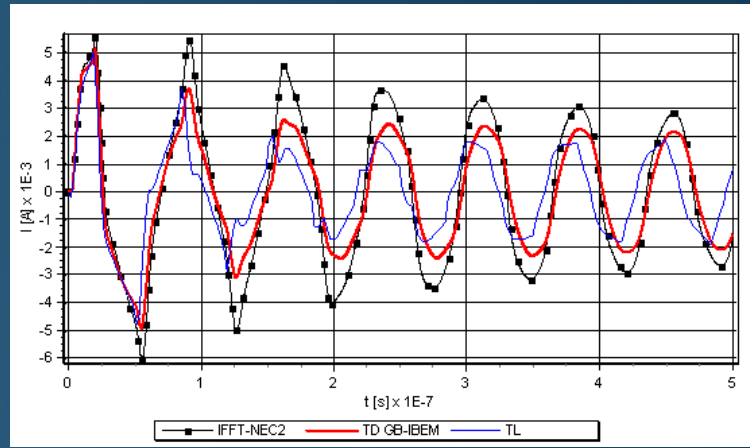
EM Field Coupling to Overhead Wires

Computational example: *Transient response of a two-wire array above a PEC ground - comparison between IFFT-NEC2, GB-IBEM and TL*

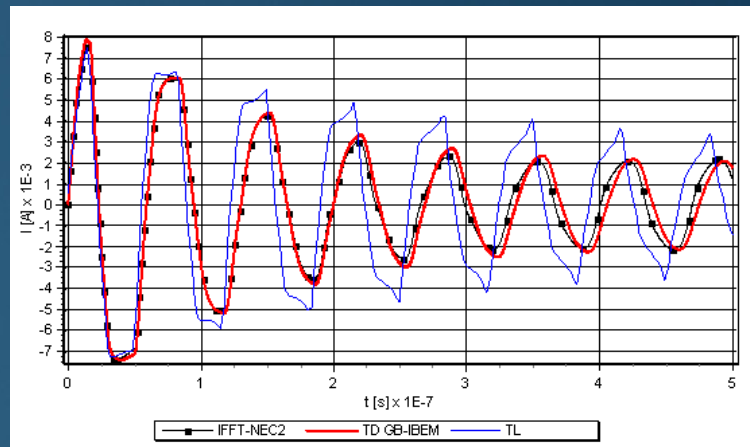


L=10m, a=2cm,
h_1=1m, h_2=2m, d=1m

Excitation: EMP
E_0=1V/m, $a=4*10^7 s^{-1}$, $b=6*10^8 s^{-1}$



Transient current induced at the center of wires 1 and 3



Transient current induced at the center of wire 2

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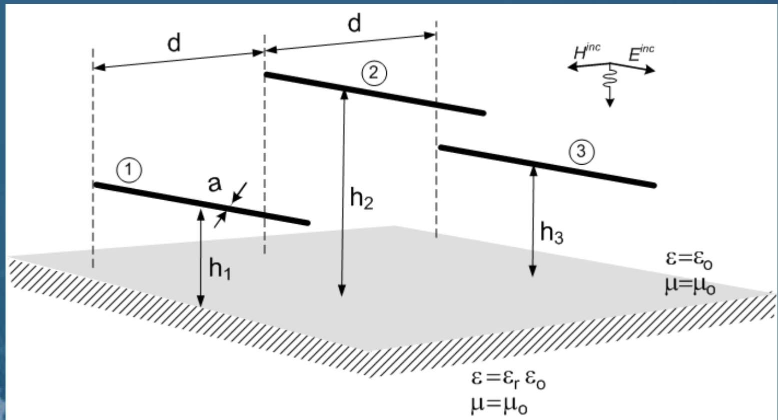
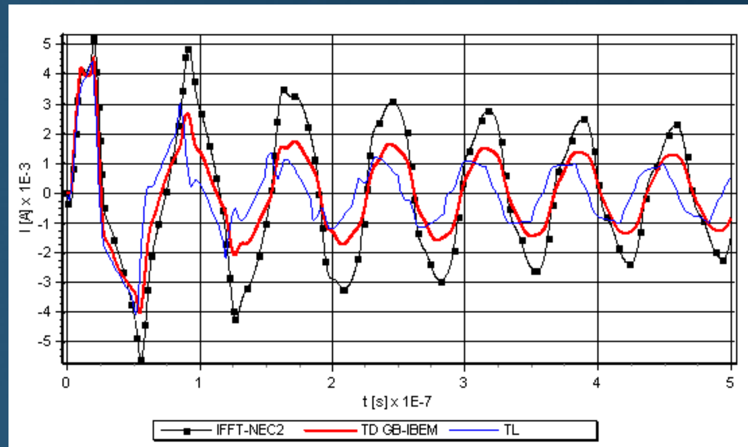
EM Field Coupling to Overhead Wires

Computational example: *Transient response of a three -wire array above a PEC ground - comparison between IFFT-NEC2, GB-IBEM and TL*

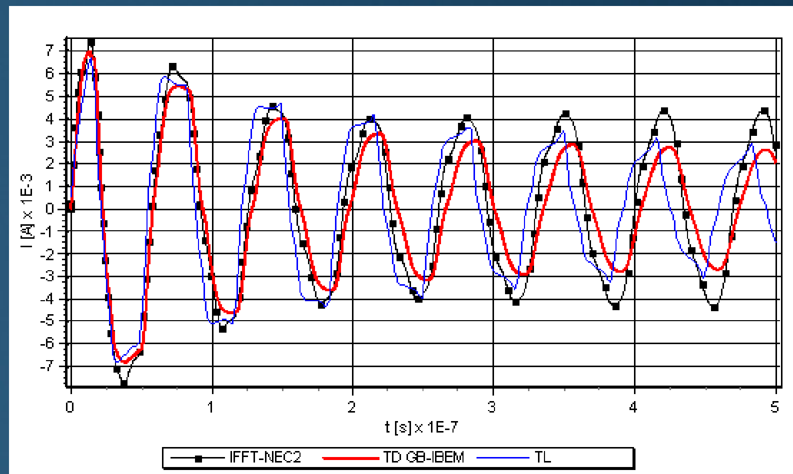




Transient current induced at the center of wires 1 and 3

L=10m, a=2cm,
$h_1=h_3$=1m, h_2=2m, d=1m
E_r=10

Excitation: EMP
E_0=1V/m, $a=4{*}10^7 s^{-1}$, $b=6{*}10^8 s^{-1}$



Transient current induced at the center of wire 2

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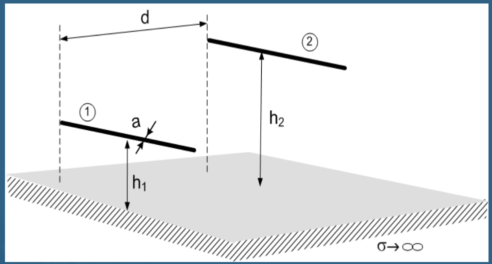
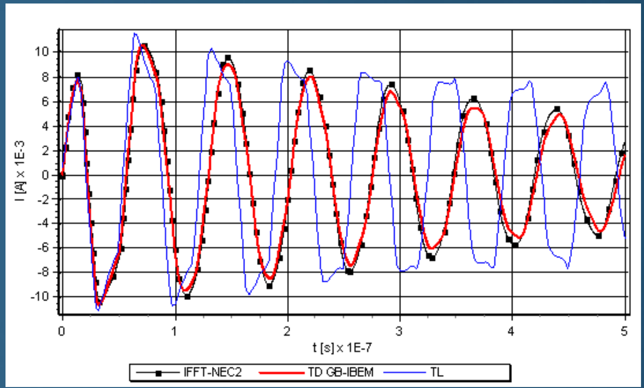
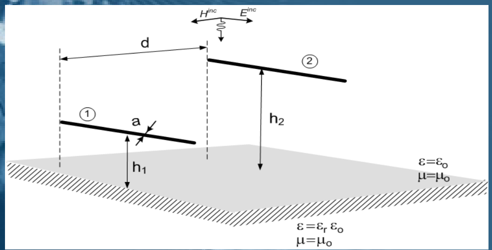
EM Field Coupling to Overhead Wires

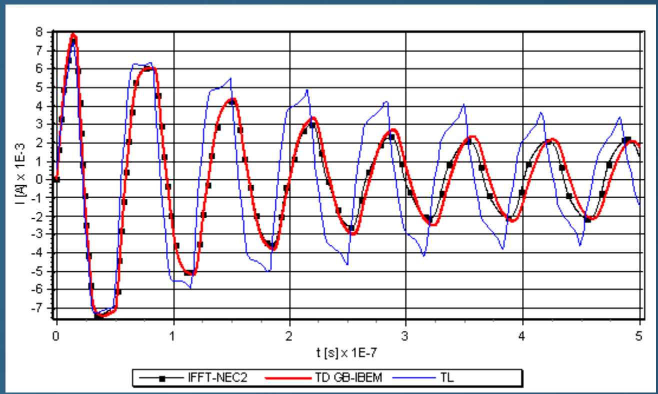
Computational example: *Two wire array: PEC and dielectric half space comparisons*





Transient current induced at the center of wire 2 – PEC ground





Transient current induced at the center of wire 2 – dielectric half-space

The results obtained via different approaches agree for early time instants in both cases.

At later times TL fails to ensure valid results due to limitations of the model itself (radiation effects).

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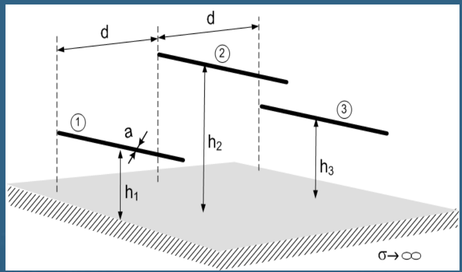
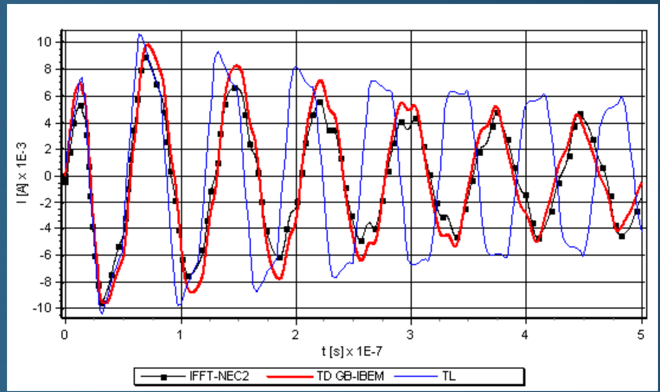
EM Field Coupling to Overhead Wires

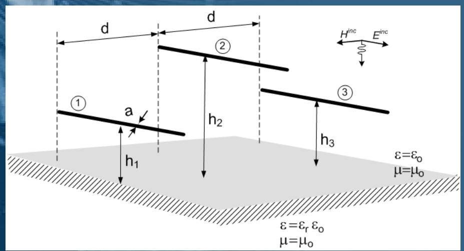
Computational example: *Two wire array: PEC and dielectric half space comparisons*

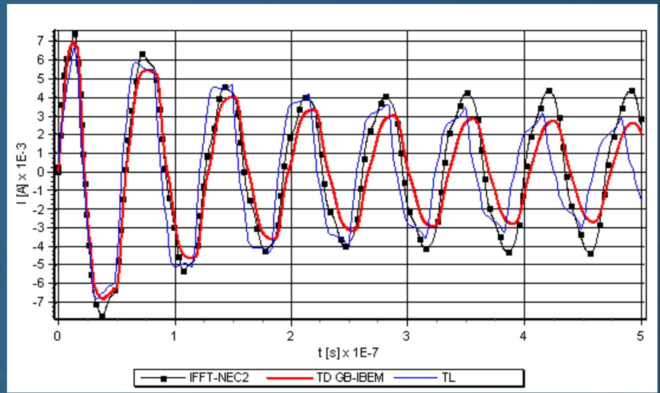




Transient current induced at the center of wire 2 – PEC ground

The behaviour of 3-wires above PEC ground is similar to a two-wire array.





Transient current induced at the center of wire 2 – dielectric half-space

For dielectric half-space NEC 2 produces non-physical solution (magnitude increase) at later times.

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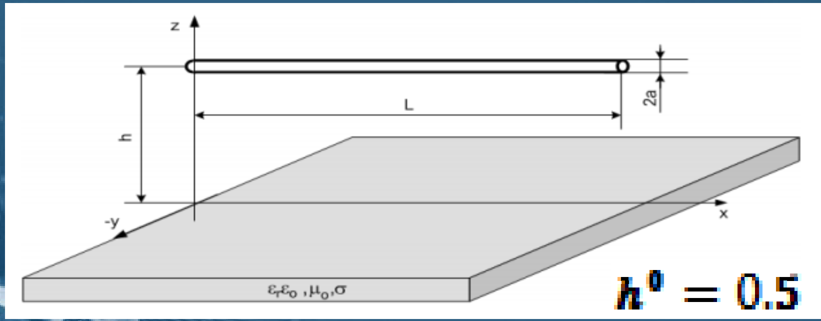
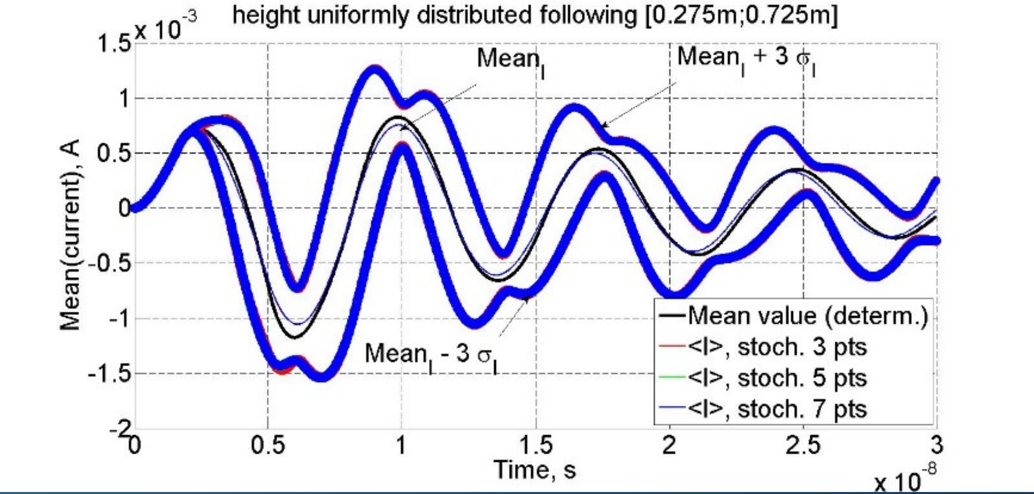
EM Field Coupling to Overhead Wires

Computational example: Transient response of GPR dipole antenna

The excitation: Gaussian pulse



Deterministic thin wires above lossy ground.



Stochastic case #2: wire height uniformly distributed between 0.275 and 0.725 m

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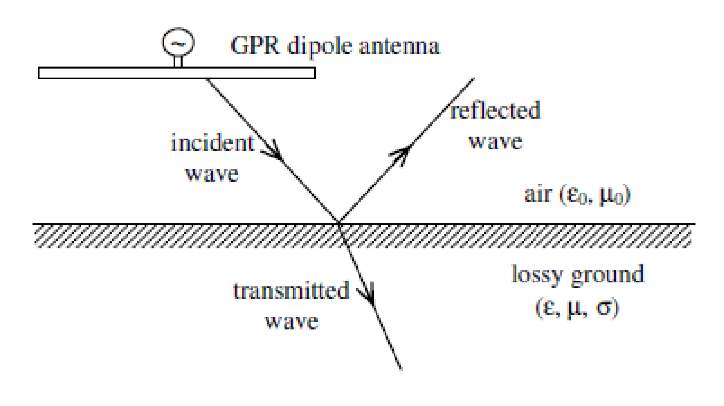
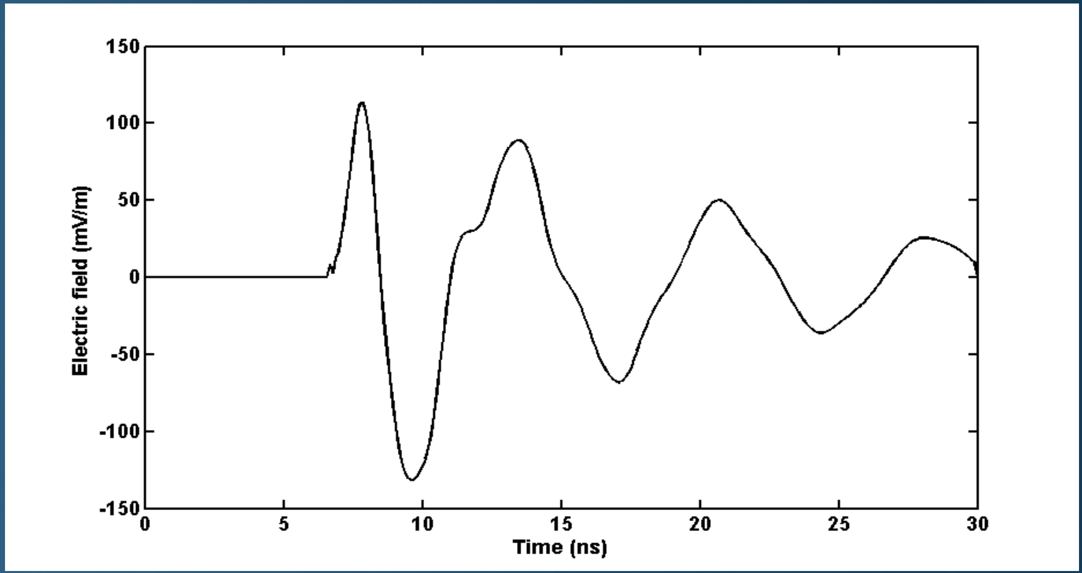
EM Field Coupling to Overhead Wires

Computational example: Transient response of GPR dipole antenna at the center of the line

The excitation: Gaussian pulse



GPR dipole antenna above a lossy half-space



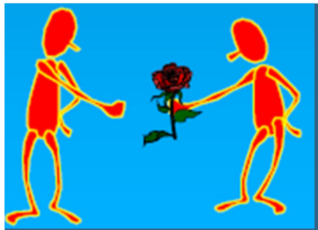
Transmitted electric field in the dielectric half-space (ε_r=10)

$$\int_0^L \frac{I(x',t-\frac{R}{c})}{4\pi R}dx' - \int_{-\infty}^{t}\int_0^L r(\theta,\tau)\frac{I(x',t-\frac{R^*}{c}-\tau)}{4\pi R^*}dx'd\tau = \frac{1}{2Z_0}\int_0^L E_x^{inc}(x',t-\frac{|x-x'|}{c})dx' + F_0(t-\frac{x}{c}) + F_L(t-\frac{L-x}{c})$$

$$E_x^{tr}(r,t) = \frac{\mu_0}{4\pi}\int_{-\infty}^{t}\int_0^L \Gamma_{tr}^{MIT}(\theta,\tau)\frac{\partial I(x',t-R''/v-\tau)}{\partial t}\frac{e^{-\frac{1}{\tau_g}\frac{R''}{v}}}{R''}dx'd\tau$$

There is scarcely a subject that cannot be mathematically treated and the effect calculated beforehand, or the results determined beforehand from the available theoretical and practical data.

Nikola Tesla



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We made models in science, but we also made them in everyday life.
STEPHEN HAWKING

Thank you for your attention