



# TWINS II

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*Training School on Ground Penetrating Radar for civil engineering and cultural  
heritage management*

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# TWINS I = Thin Wire Numerical Solver

Suzana = Sustav za analizu antena (Croatian)

- Used for educational purpose and scientific research

## What is solved?

- Governing equations for a current distribution along the wire radiating above a half space

Frequency domain solver (FD)



Time domain solver (TD)

## Numerical method?

- Galerkin-Bubnov scheme of Indirect Boundary Element Method (GB-IBEM)

## Programming language:

- C++

## TD vs. FD techniques

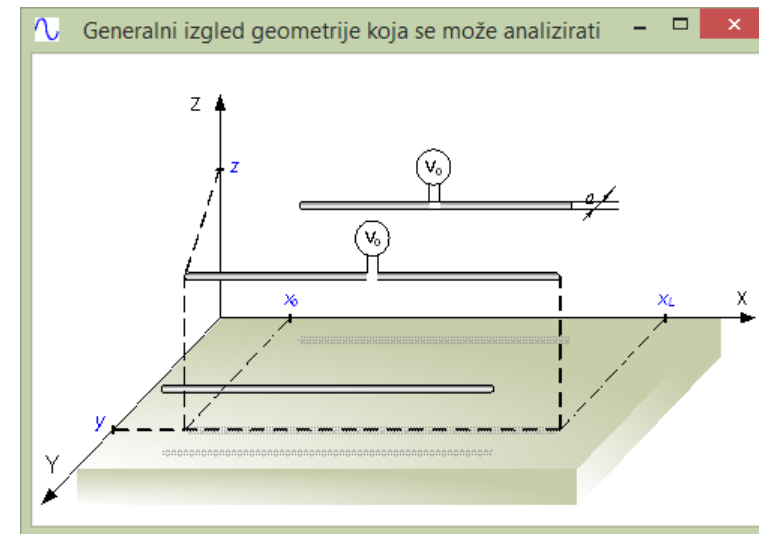
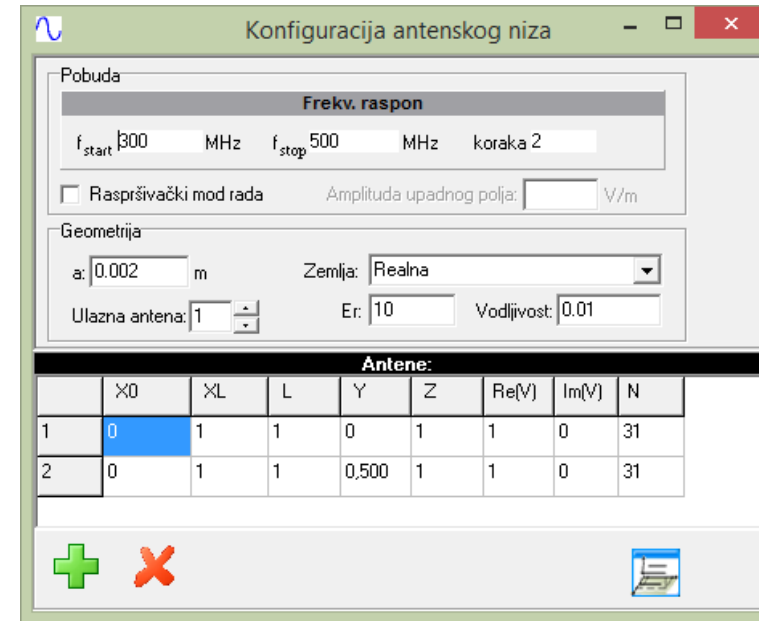
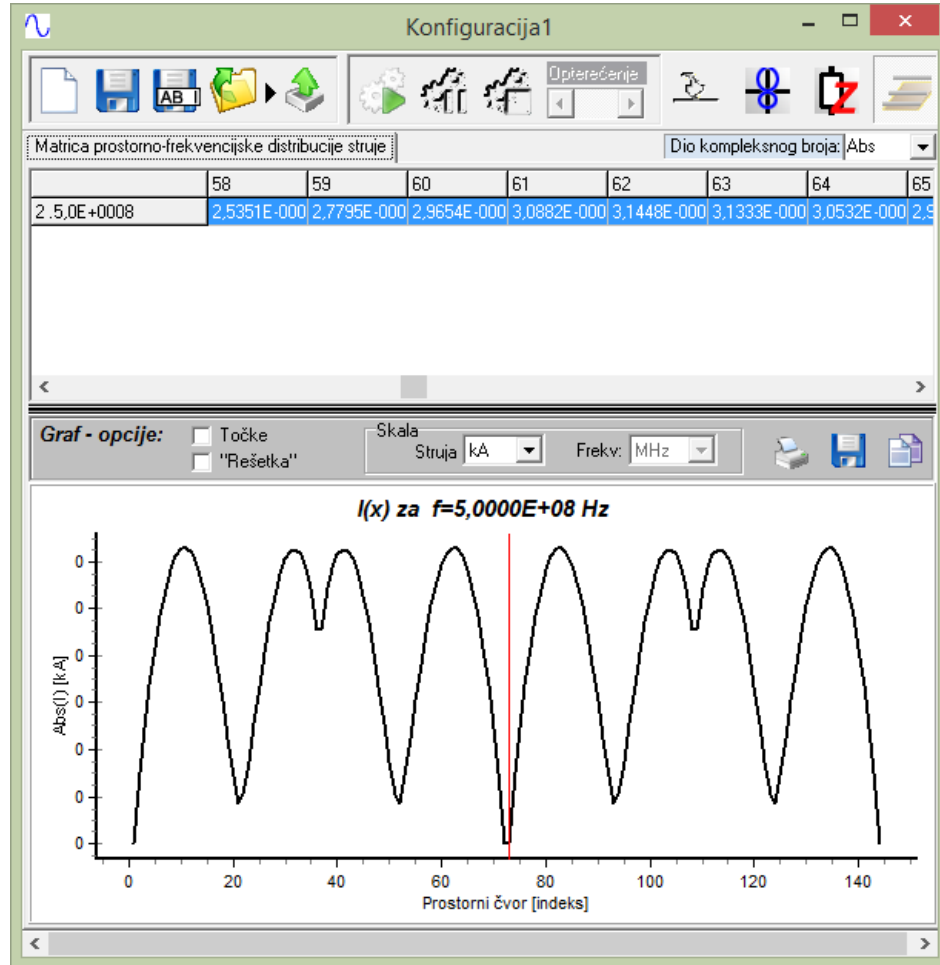
### Time domain (TD) techniques:

- Applied for a large frequency spectrum, but for single source
- More complex formulation requiring more computational effort
- Deeper physical insight

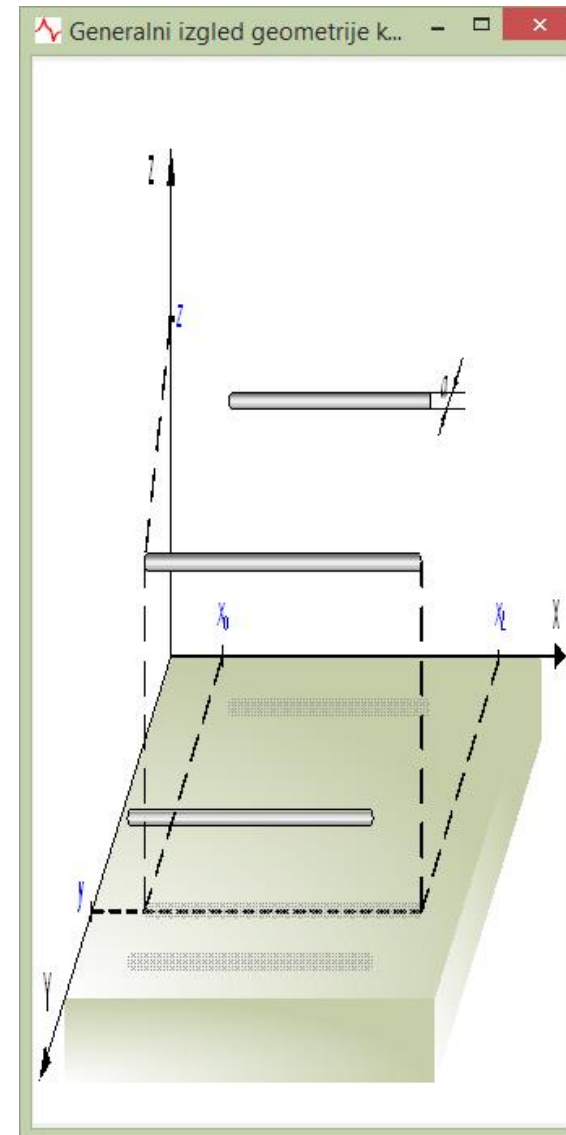
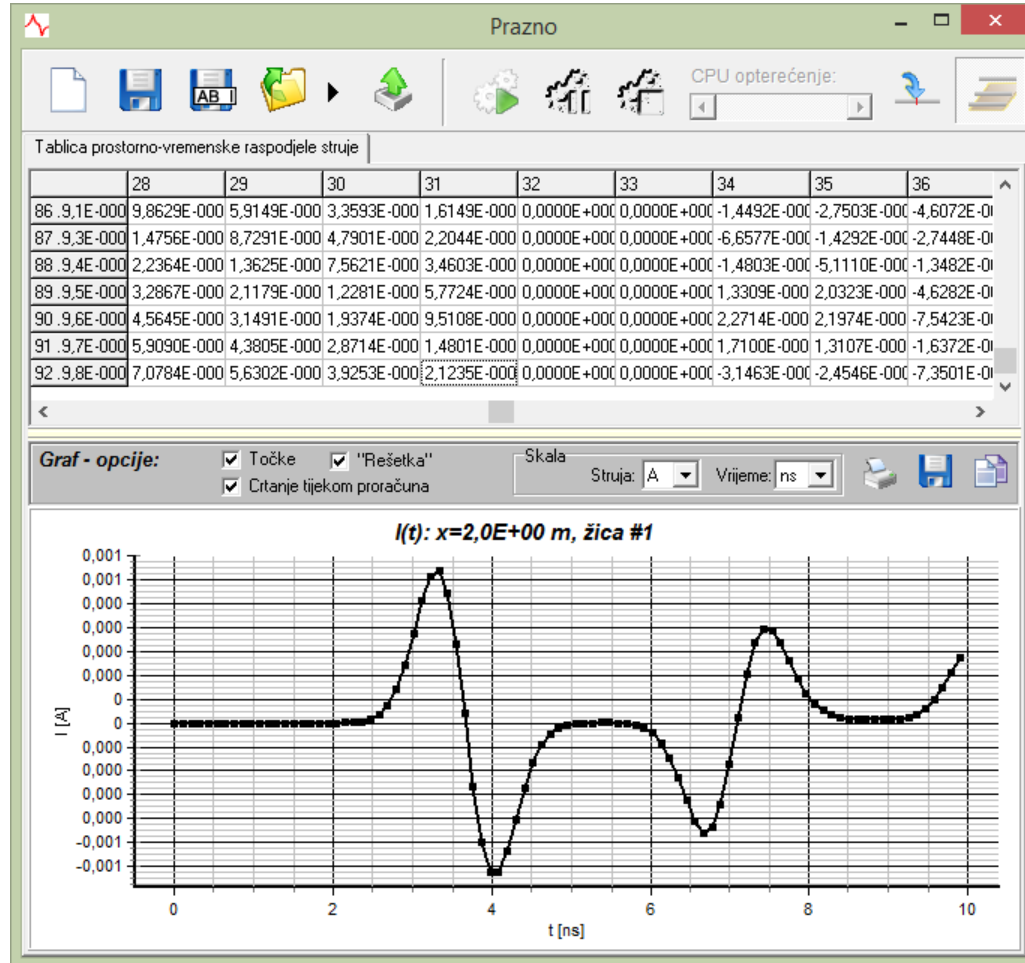
### Frequency domain (FD) techniques:

- Applied for more sources, but at a single frequency
- Formulation is substantially less demanding
- In general, regulatory standards are specified in FD form; convenient for analyzing EMC properties

# TWINS I – Frequency Domain (FD)



# TWINS I – Time Domain (TD)



Konfiguracija antenskog niza

a: 2E-3 m

T: 10E-9 s

Er: 10

Savršeno vodljiva zemlja

Pobuda:  Rasprivač Aktivna žica: 1

Pobudna fc.: Gaussov puls

Antene:						
	X0	XL	L	Y	Z	N
*1	1	2	1	5	1	31
2	1	2	1	5,50	1	31

# TWINS II

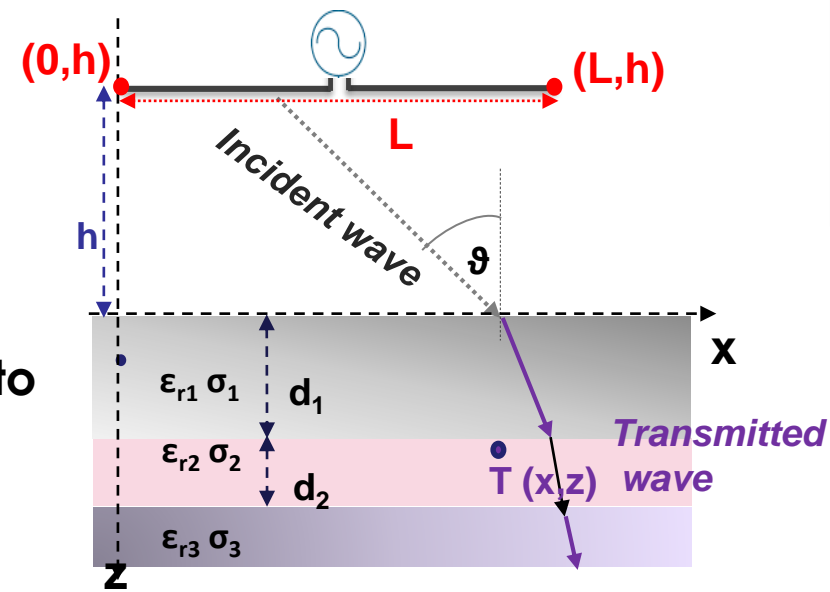
- Frequency Domain (FD)
- Presented on final GPR conference in Warsaw

## What is solved?

Integral formulas for the electric field transmitted into the subsurface.

## NEW!

The subsurface is **non-homogenous**.



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Field transmitted into ground: frequency domain integral equation approach

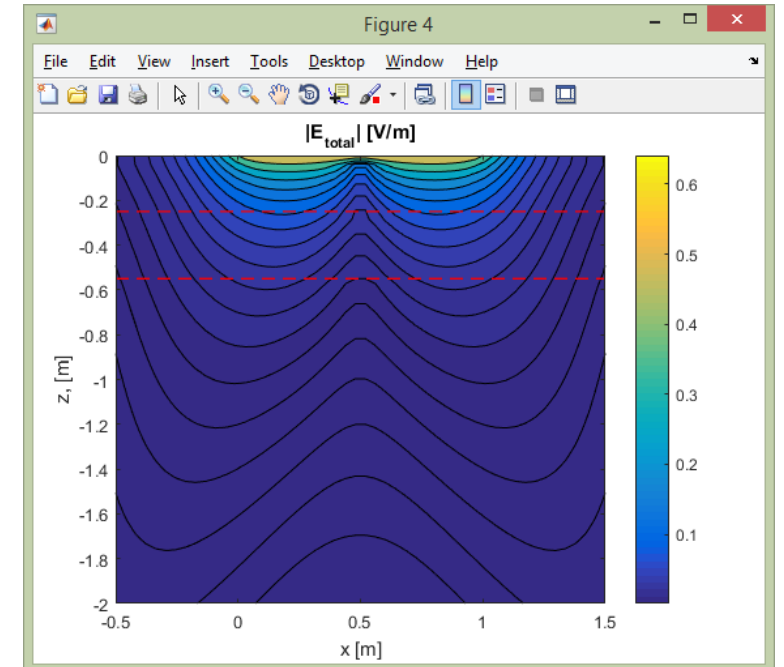
Antenna parameters: length 1, radius (mm) 6.74, height (cm) 10

Ground parameters: Layer 1 (permittivity 10, conductivity 0, thickness 0.25), Layer 2 (permittivity 8, conductivity 0, thickness 0.3), Layer 3 (permittivity 10, conductivity 0, thickness 0)

Run the simulation: Method MIT (selected), RCA (selected). Run the simulation, Save

Frequency domain (FD): Single frequency (selected), Single point, xz grid, multiple F. Coordinates (m): x axis (START 0.5, END 0.5, No. points 1), z axis (START 0, END 2, No. points 50). Frequency (MHz): 100, 150, 15

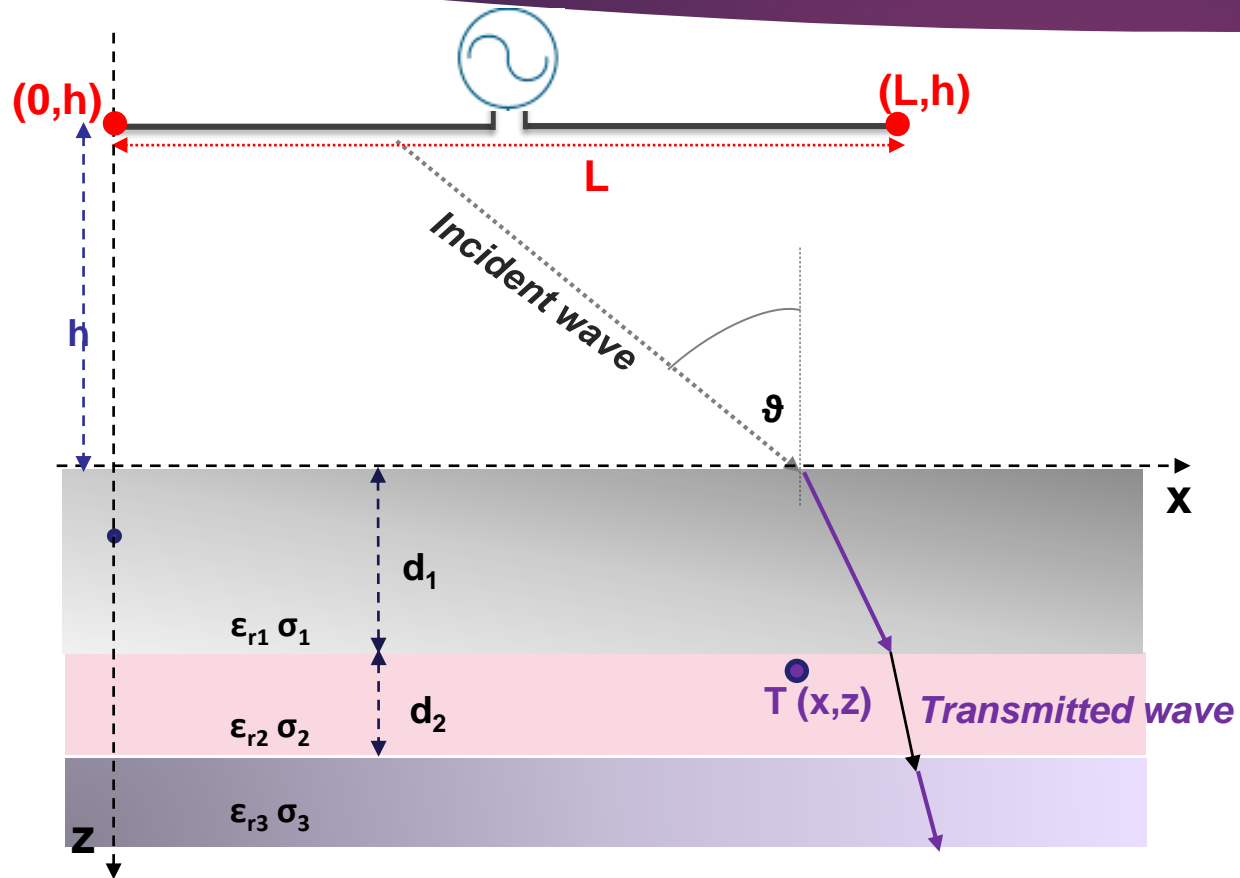
Time Domain (TD): Inverse Fast Fourier Transform (IFFT) to TD. time delay (ns) 1.43, time frame (ns) 40, halwidth (ns) 0.667, x (m) 0.5, z (m) 0.5. Show Gauss pulse, Save frequency domain field



# Talk Layout

- **THEORETICAL BACKGROUND**
  - Model geometry
- **INSTALLATION PROCEDURE**
- **USER GUIDE:**
  - Antenna parameters
  - Ground parameters
  - Frequency domain (FD)
  - Time domain (TD)
  - Run the simulation
- **EXAMPLES**
- **REFERENCES**

# Model geometry



Ground penetrating radar dipole antenna above a lossy half space.

The half space is modeled as a three layered medium.



## Pocklington's integro-differential equation:

$$E_x^{exc} = j \omega \frac{\mu}{4\pi} \int_{-\frac{L}{2}}^{\frac{L}{2}} I(x') g(x, x') dx' - \frac{1}{j4\pi\omega\epsilon_0} \frac{\partial}{\partial x} \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{\partial I(x')}{\partial x'} g(x, x') dx'$$

$I(x')$  ... the unknown current

$g(x, x')$  ... Green's function

$$g(x, x') = g_0(x, x') - \mathbf{R}_{TM} g_i(x, x')$$

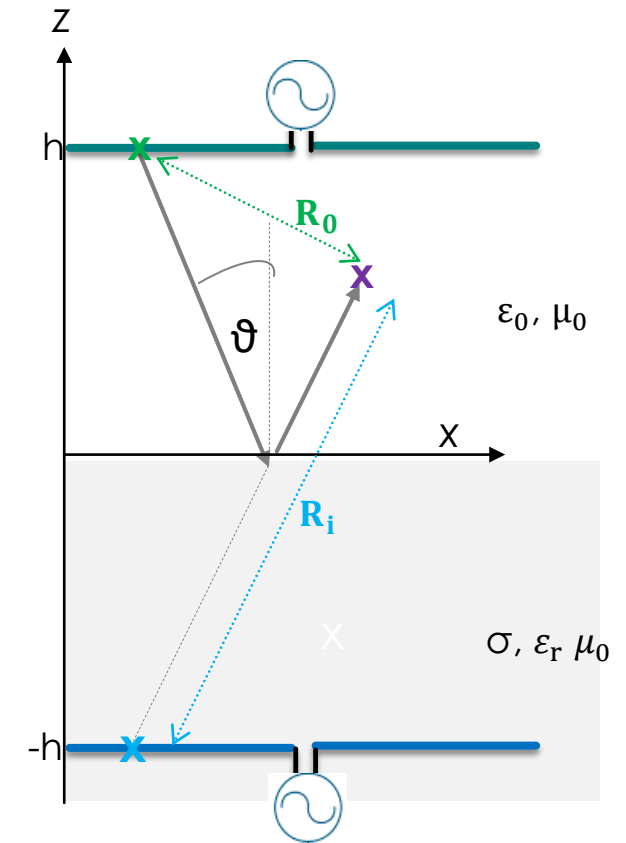
$g_0(x, x')$  ... for free space

$$g_0(x, x') = \frac{e^{-jk_0 R_0}}{R_0}$$

$g_i(x, x')$  ... from image theory

$$g_i(x, x') = \frac{e^{-jk_0 R_i}}{R_i}$$

Wire above a lossy half-space:



## Electric field integral relations:

$$E_x = \frac{1}{j4\pi\omega\epsilon_0} \left[ \int_0^L \frac{\partial I(x')}{\partial x'} \frac{\partial \mathbf{g}(x, \mathbf{y}, \mathbf{z}, x')}{\partial x} dx' - \gamma^2 \int_0^L I(x') \mathbf{g}(x, x') dx' \right]$$

$$E_y = \frac{1}{j4\pi\omega\epsilon_0} \int_0^L \frac{\partial I(x')}{\partial x'} \frac{\partial \mathbf{g}(x, \mathbf{y}, \mathbf{z}, x')}{\partial y} dx'$$

$$E_z = \frac{1}{j4\pi\omega\epsilon_0} \int_0^L \frac{\partial I(x')}{\partial x'} \frac{\partial \mathbf{g}(x, \mathbf{y}, \mathbf{z}, x')}{\partial z} dx'$$

$$\mathbf{g}(x, x') = \mathbf{T}_{TM} g_0(x, x')$$

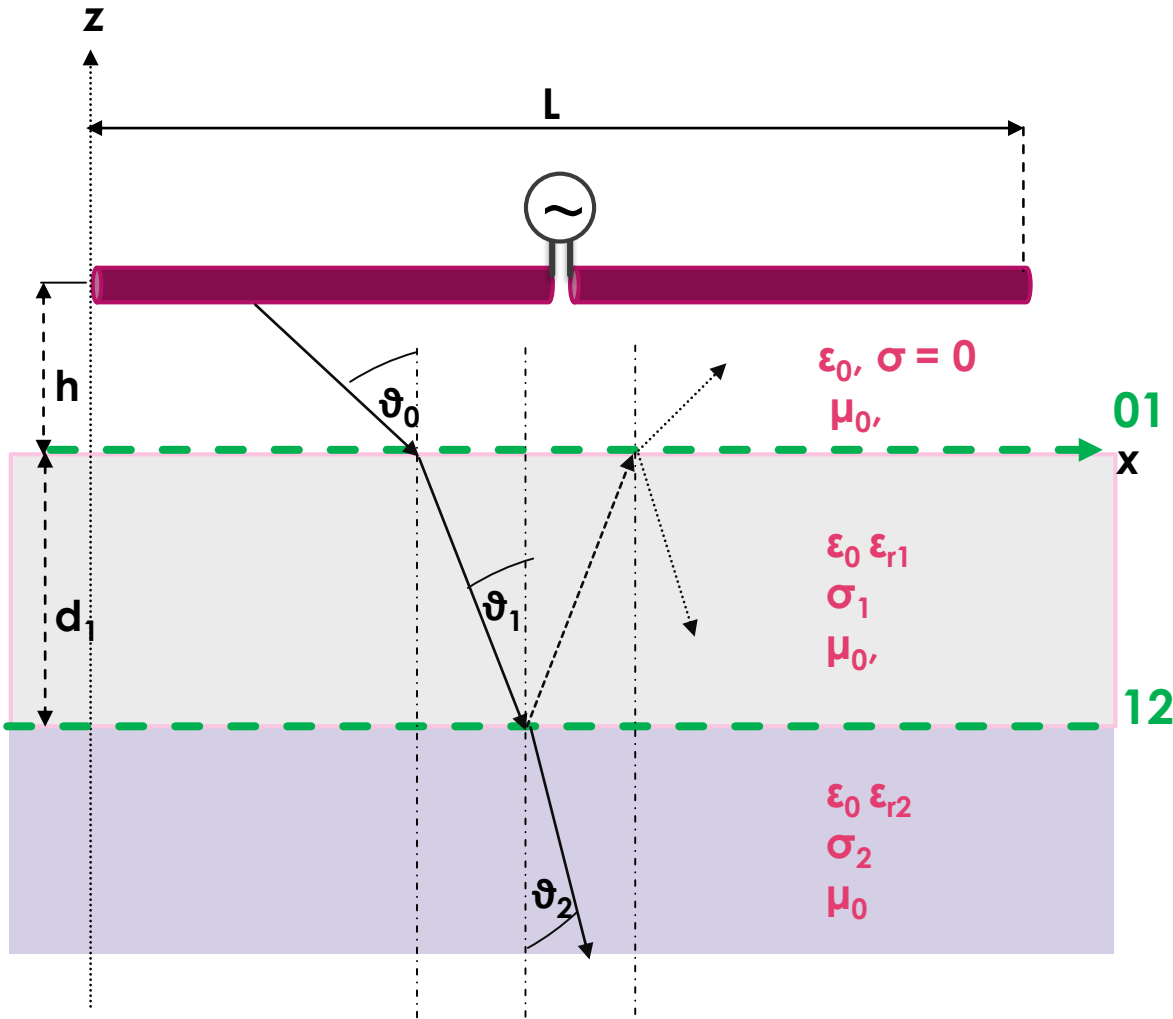
## Numerical procedure:

Galerkin Bubnov scheme of Indirect Boundary Element Method (GB-IBEM)

# Reflection/transmission coefficients for three layered media configuration:

Air + Layer1 + Layer2

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The **plane wave** approximation with **oblique incidence** at interfaces.

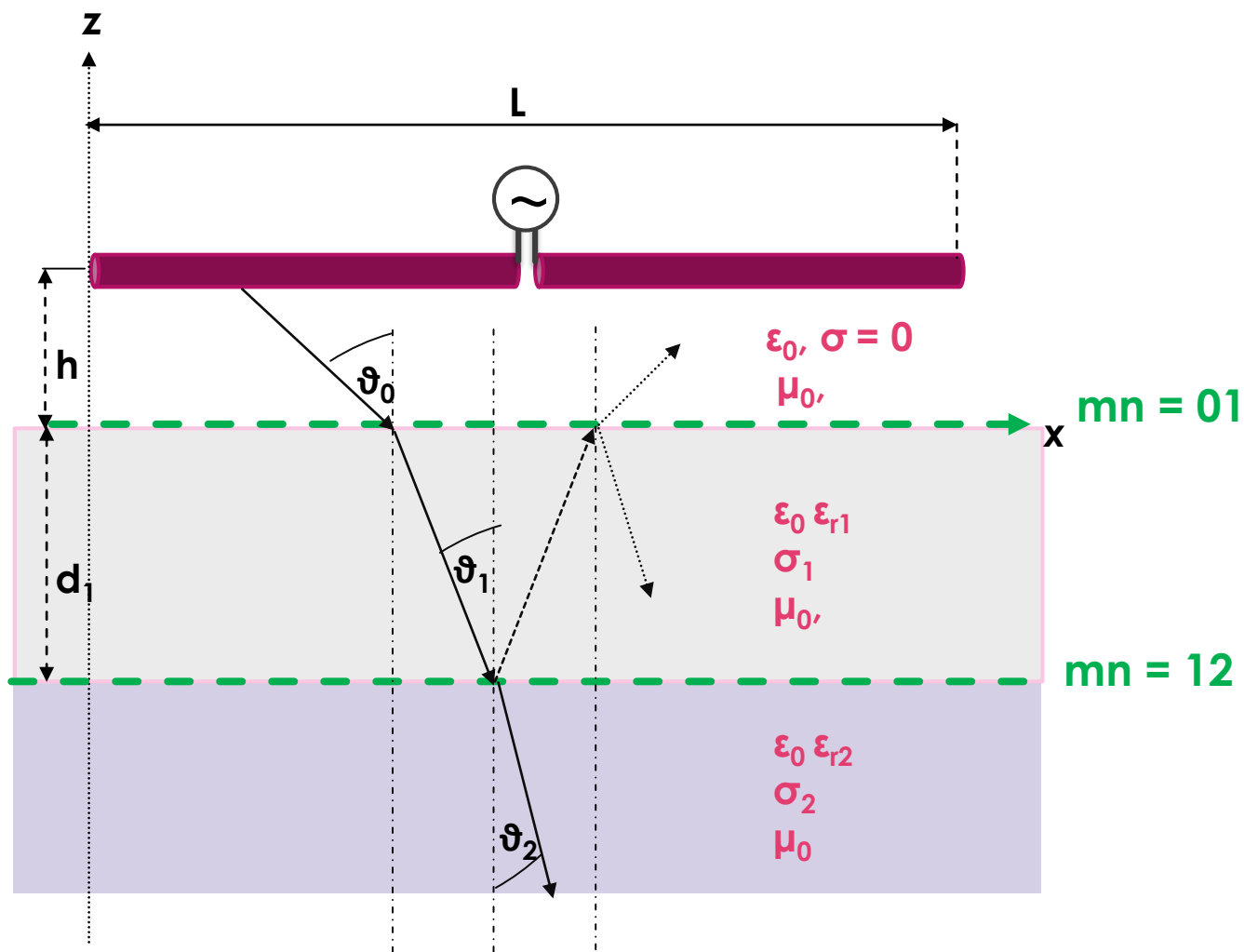
The **continuity conditions** for the tangential components of electric and magnetic fields at two interfaces (**01** and **12**) and the **Snell's law** for wave propagation yield the following equations

$$E_0^+ = \frac{1}{\tau_{01}} E_1^+ + \frac{\rho_{01}}{\tau_{01}} E_1^- \quad E_1^+ = \frac{1}{\tau_{12}} e^{\gamma_1 \cos \vartheta_1 d_1} E_2^+$$

$$E_0^- = \frac{\rho_{01}}{\tau_{01}} E_1^+ + \frac{1}{\tau_{01}} E_1^- \quad E_1^- = \frac{\rho_{12}}{\tau_{12}} e^{-\gamma_1 \cos \vartheta_1 d_1} E_2^+$$

$E_0^+, E_1^+, E_0^-$  &  $E_1^-$  ...the magnitude of electric field at the layer interfaces for the wave propagating in the negative and positive direction of  $z$  axis, respectively

# Fresnel's reflection/transmission coefficient approximation at interfaces



$$\rho_{mn} = \frac{Z_n \cos \vartheta_n - Z_m \cos \vartheta_m}{Z_n \cos \vartheta_n + Z_m \cos \vartheta_m}$$

$$\tau_{mn} = \frac{2Z_n \cos \vartheta_m}{Z_n \cos \vartheta_n + Z_m \cos \vartheta_m}$$

$$m = 0,1 \quad n = 1,2$$

Impedance of medium

complex propagation constant

$$Z_k = \sqrt{\frac{j\omega\mu_0}{\sigma_k + j\omega\epsilon_0\epsilon_{rk}}}$$

$$\gamma_k = \sqrt{j\omega\mu_0(\sigma_k + j\omega\epsilon_0\epsilon_{rk})} \quad k = 0,1,2$$

$$\begin{bmatrix} E_0^+ \\ E_0^- \end{bmatrix} = M_{01} \begin{bmatrix} E_1^+ \\ E_1^- \end{bmatrix} \quad \begin{bmatrix} E_1^+ \\ E_1^- \end{bmatrix} = P_{12} M_{12} \begin{bmatrix} E_2^+ \\ E_2^- \end{bmatrix}$$

... Matrix form for equations that solve for field values at layer's interfaces

$$M_{mn} = \frac{1}{\tau_{mn}} \begin{bmatrix} 1 & \rho_{mn} \\ \rho_{mn} & 1 \end{bmatrix} \quad m = 0,1, n = 1,2$$

... matching matrix

$$P_{12} = \begin{bmatrix} e^{\gamma_1 \cos \vartheta_1 d_1} & 0 \\ 0 & e^{-\gamma_1 \cos \vartheta_1 d_1} \end{bmatrix}$$

... propagation matrix

The layers  $k = 3, \dots$  are readily included by adding the propagation and matching matrices :

By setting  $E_0^+ = 1 \text{ V/m}$ , all the other magnitudes  $E_m^+$  and  $E_m^-$  are easily determined.

Electric field can be calculated using the following expressions:

$$\vec{E}_0 = E_1^+ (\cos \vartheta_0 \vec{e}_x + \sin \vartheta_0 \vec{e}_z) e^{-\gamma_0(x \sin \vartheta_0 - z \cos \vartheta_0)} + E_0^- (\cos \vartheta_0 \vec{e}_x - \sin \vartheta_0 \vec{e}_z) e^{-\gamma_0(x \sin \vartheta_0 + z \cos \vartheta_0)}$$

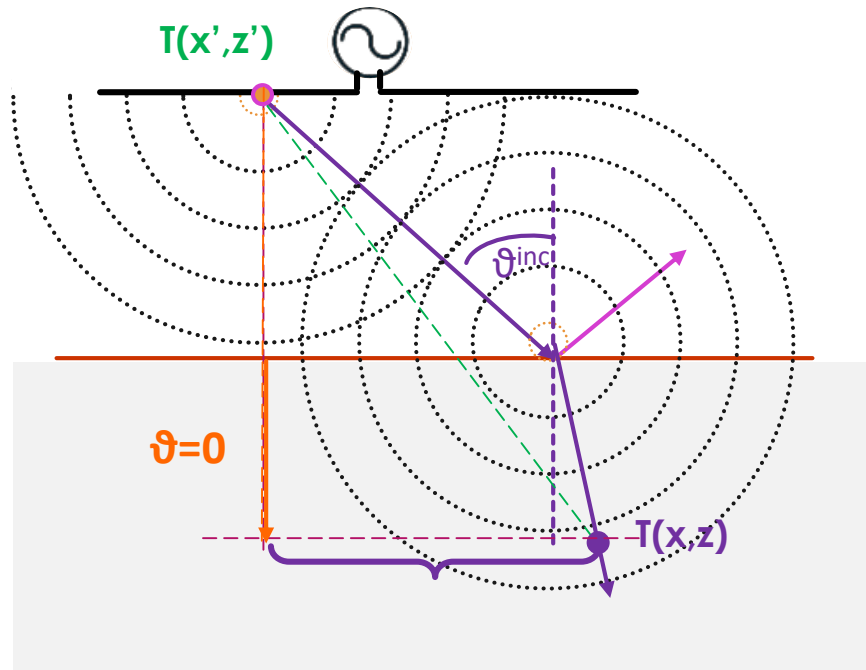
$$\vec{E}_1 = E_1^+ (\cos \vartheta_1 \vec{e}_x + \sin \vartheta_1 \vec{e}_z) e^{-\gamma_1(x \sin \vartheta_1 - z \cos \vartheta_1)} + E_1^- (\cos \vartheta_1 \vec{e}_x - \sin \vartheta_1 \vec{e}_z) e^{-\gamma_1(x \sin \vartheta_1 + z \cos \vartheta_1)}$$

$$\vec{E}_2 = E_2^+ (\cos \vartheta_2 \vec{e}_x + \sin \vartheta_2 \vec{e}_z) e^{-\gamma_2(x \sin \vartheta_2 - z \cos \vartheta_2)}$$

The reflection and transmission coefficients  $\mathbf{R}_{\text{TM}}$  and  $\mathbf{T}_{\text{TM}}$  appearing in the Green's function are calculated as the ratio between the field value at the ground surface and the field value obtained from the equations above at the given observation point.

## Alternative simplified approach to calculate transmission / reflection coefficients:

- Modified image theory based approach
- convenient for TD computations
- a comparison with the plane wave approximation in FD needed for benchmark



The **MIT** approach assumes the **normal incidence** of a propagating EM wave, therefore the incidence angles are zero:  $\vartheta_m = 0^\circ$ .

The expressions for the reflection and transmission coefficients for the two adjacent media indexed with  $m$  and  $n$  are given as:

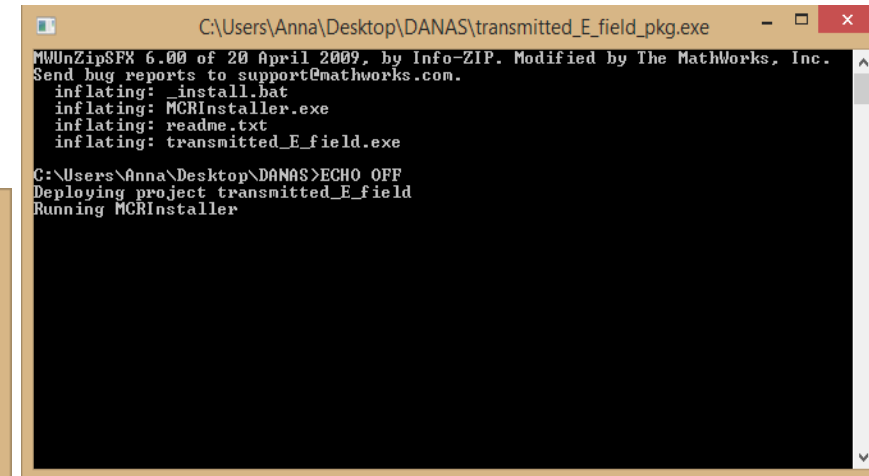
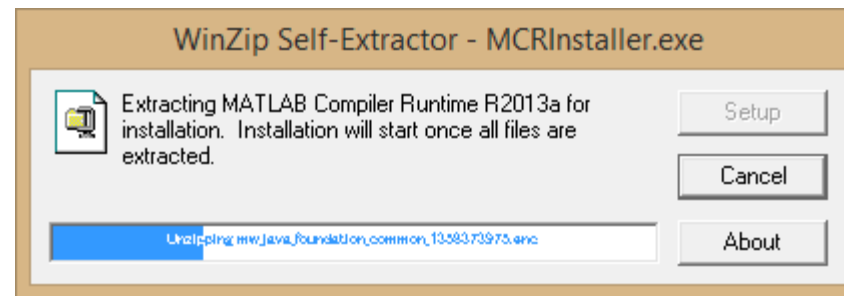
$$\rho_{mn}^{MIT} = \frac{(Z_m^2 - Z_n^2)}{(Z_m^2 + Z_n^2)}$$

$$\tau_{mn}^{MIT} = \frac{2Z_m^2}{(Z_m^2 + Z_n^2)}$$

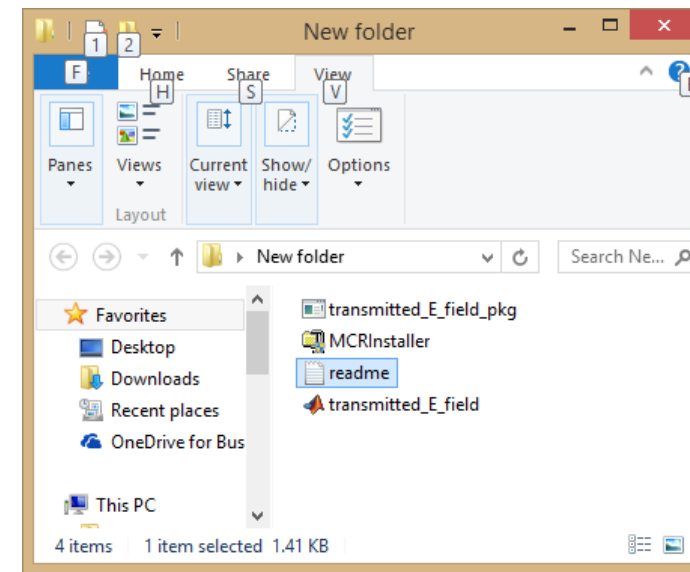
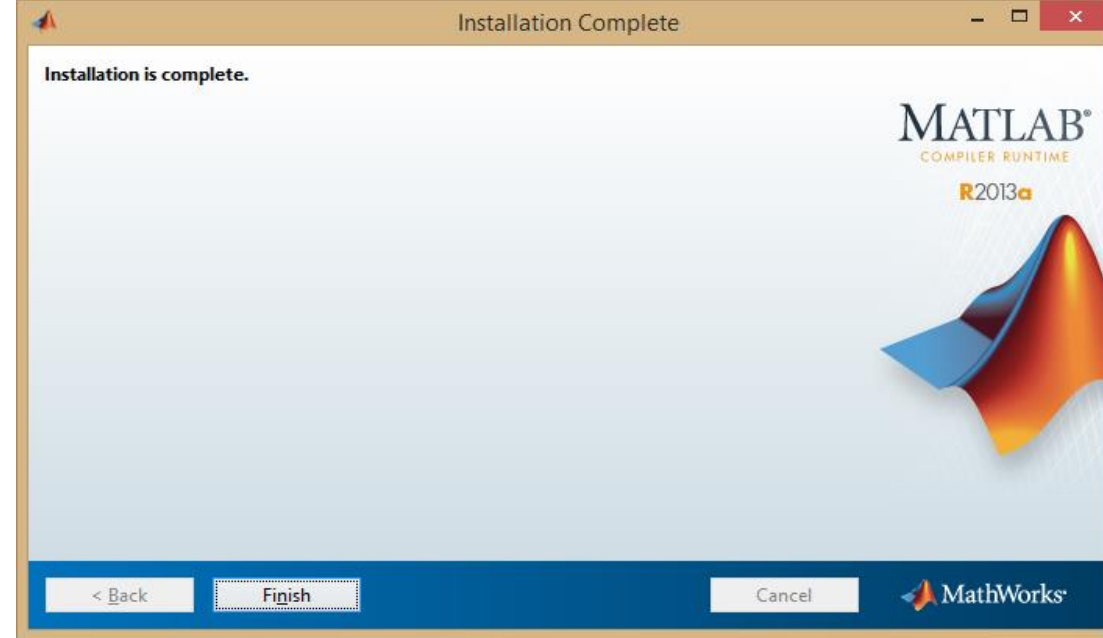
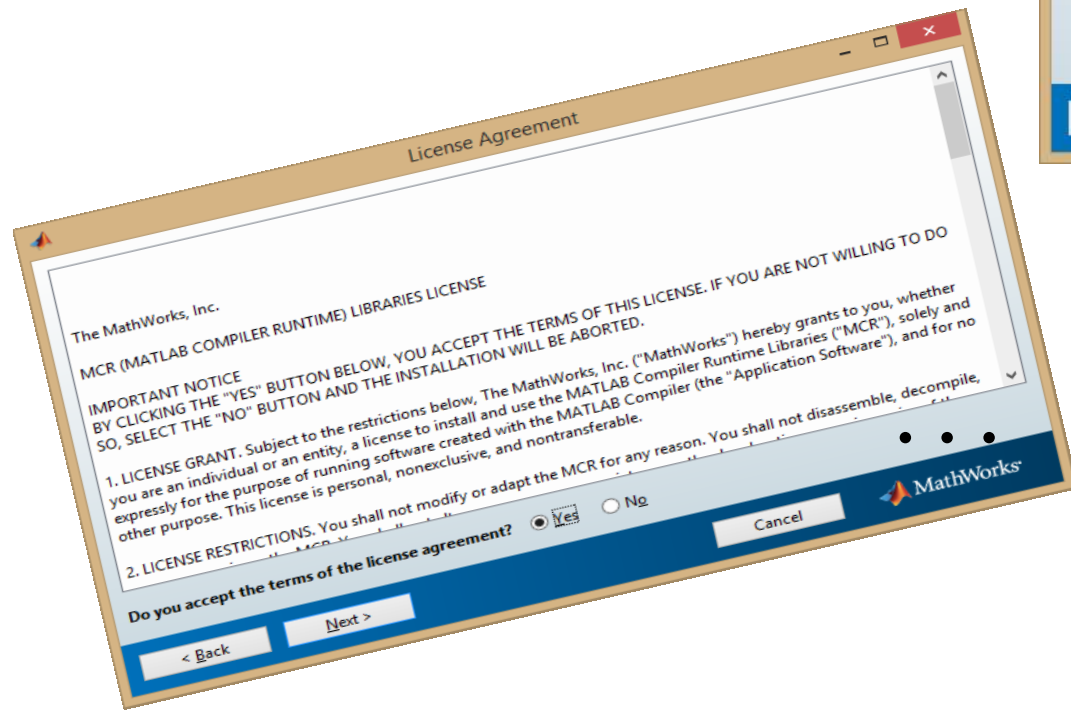
$$m = 0,1 \quad n = 1,2$$

# Installation procedure

- ▶ Run **transmitted\_E\_field\_pkg.exe**.
- ▶ This action installs MATLAB Compiler Runtime (MCR) version 8.1. and extracts the following files:
  - **transmitted\_E\_field.exe**
  - **MCRInstaller.exe**
  - **readme.txt** file







# User guide

- ANTENNA PARAMETERS
- GROUND PARAMETERS
- FREQUENCY DOMAIN (FD)
- TIME DOMAIN (TD)
- RUN THE SIMULATION

## Graphical user interface

Field transmitted into ground: frequency domain integral equation approach

**1.** Antenna parameters

length (m)  radius (mm)  height (cm)

**2.** Ground parameters

Layer 1  Layer 2  Layer 3

permittivity	<input type="text" value="10"/>	<input type="text" value="10"/>	<input type="text" value="10"/>
conductivity (mS/m)	<input type="text" value="0"/>	<input type="text" value="0"/>	<input type="text" value="0"/>
thickness (cm)	<input type="text" value="25"/>	<input type="text" value="30"/>	<input type="text"/>

**3.** Frequency domain (FD)

Single frequency  Single point  xz grid, multiple F

Coordinates (m)	START	END	No. points
x axis	<input type="text" value="-0.5"/>	<input type="text" value="1.5"/>	<input type="text" value="50"/>
z axis	<input type="text" value="0"/>	<input type="text" value="2"/>	<input type="text" value="50"/>
Frequency (MHz)	<input type="text" value="10"/>	<input type="text" value="150"/>	<input type="text" value="15"/>

**4.** Time Domain (TD)

Inverse Fast Fourier Transform (IFFT) to TD

time delay (ns)	<input type="text" value="1.43"/>	time frame (ns)	<input type="text" value="40"/>
halwidth (ns)	<input type="text" value="0.667"/>	x (m)	<input type="text" value="0.5"/>
		z (m)	<input type="text" value="0.5"/>

Save frequency domain field

**5.** Run the simulation

Method:  MIT  RCA

Results are saved in:

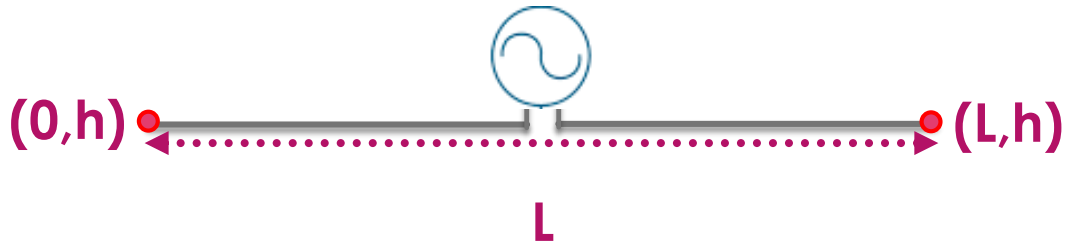
FD:

TD:

## Antenna parameters

Antenna parameters

length (m)  radius (mm)  height (cm)

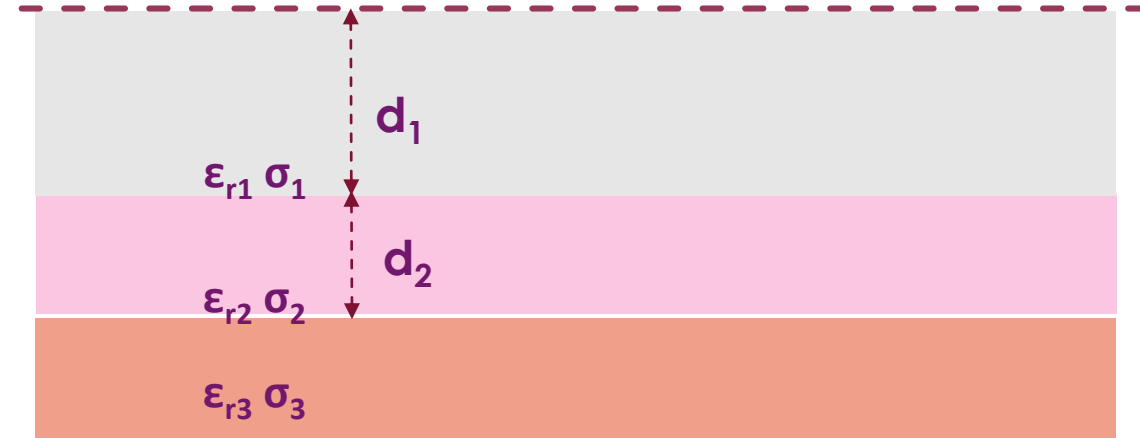


## Ground parameters

Ground parameters

Layer 1  Layer 2  Layer 3

permittivity	<input type="text" value="10"/>	<input type="text" value="10"/>	<input type="text" value="10"/>
conductivity (mS/m)	<input type="text" value="0"/>	<input type="text" value="0"/>	<input type="text" value="0"/>
thickness (cm)	<input type="text" value="25"/>	<input type="text" value="30"/>	<input type="text"/>



# Frequency domain (FD)

Frequency domain (FD)

Single frequency     Single point     xz grid, multiple F

Coordinates (m)	START	END	No. points
x axis	-0.5	1.5	50
z axis	0	2	50
Frequency (MHz)	10	150	15

Frequency domain (FD)

Single frequency     Single point     xz grid, multiple F

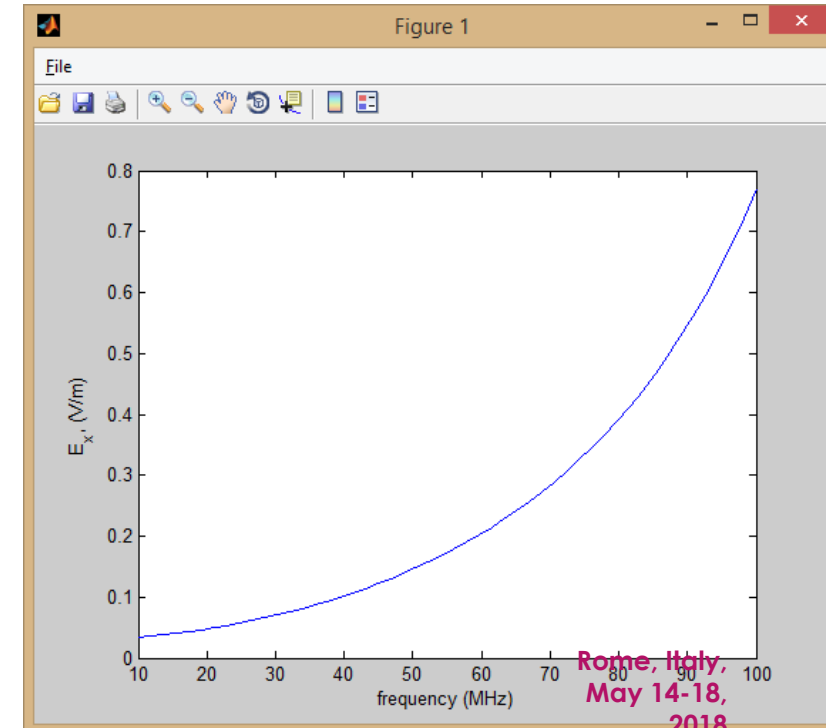
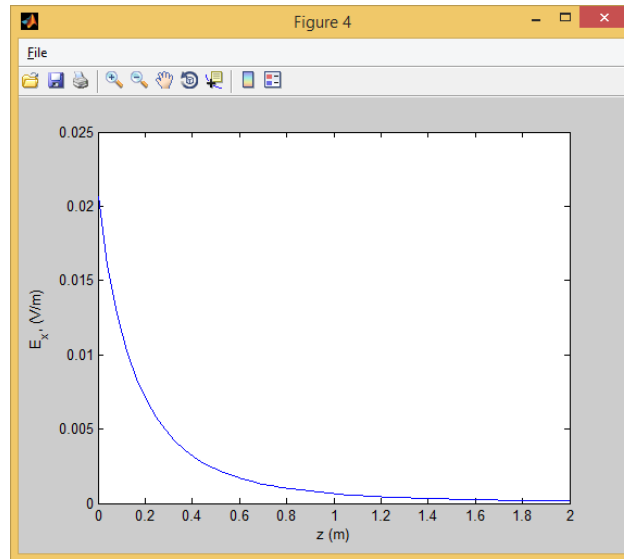
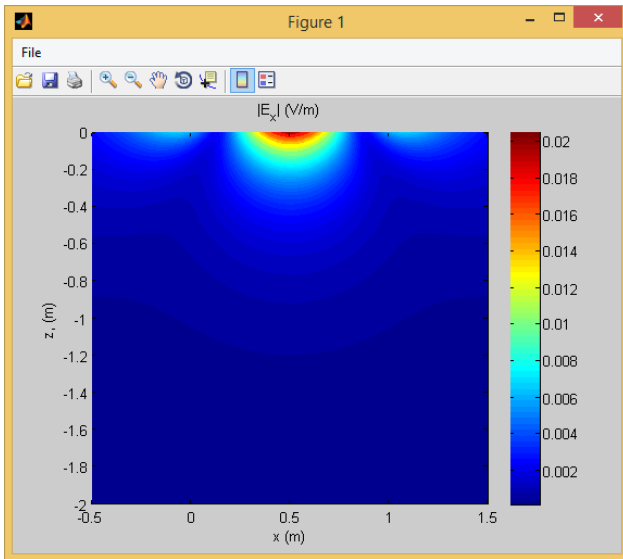
Coordinates (m)	START	END	No. points
x axis	0.5	1.5	50
z axis	0.25	2	50
Frequency (MHz)	10	100	50

Figures generated:

- **E vs. x (z)** if only one z (x) point is defined
- **contours** if xz grid is defined

Figure generated:

- **E vs. frequency**



Frequency domain (FD)

Single frequency     Single point     xz grid, multiple F

Coordinates (m)	START	END	No. points
x axis	-0.5	1.5	50
z axis	0	2	50
Frequency (MHz)	10	100	50

Figure generated:

- None
- The results are saved in .txt file

Freq. (MHz)      x(m)      z(m)      Real( $E_x$ )      Imag( $E_x$ )      Real( $E_z$ )      Imag( $E_z$ )

Freq. (MHz)	x(m)	z(m)	Real( $E_x$ )	Imag( $E_x$ )	Real( $E_z$ )	Imag( $E_z$ )
10000000.000000	-0.500000	0.000000	-0.001053	-0.002397	0.000729	0.000839
10000000.000000	-0.459184	0.000000	-0.001170	-0.002655	0.000862	0.001014
10000000.000000	-0.418367	0.000000	-0.001306	-0.002949	0.001029	0.001236
10000000.000000	-0.377551	0.000000	-0.001466	-0.003287	0.001240	0.001519
10000000.000000	-0.336735	0.000000	-0.001650	-0.003671	0.001510	0.001883
10000000.000000	-0.295918	0.000000	-0.001860	-0.004104	0.001855	0.002354
10000000.000000	-0.255102	0.000000	-0.002089	-0.004582	0.002301	0.002966
10000000.000000	-0.214286	0.000000	-0.002327	-0.005089	0.002876	0.003761

# Time domain (TD)

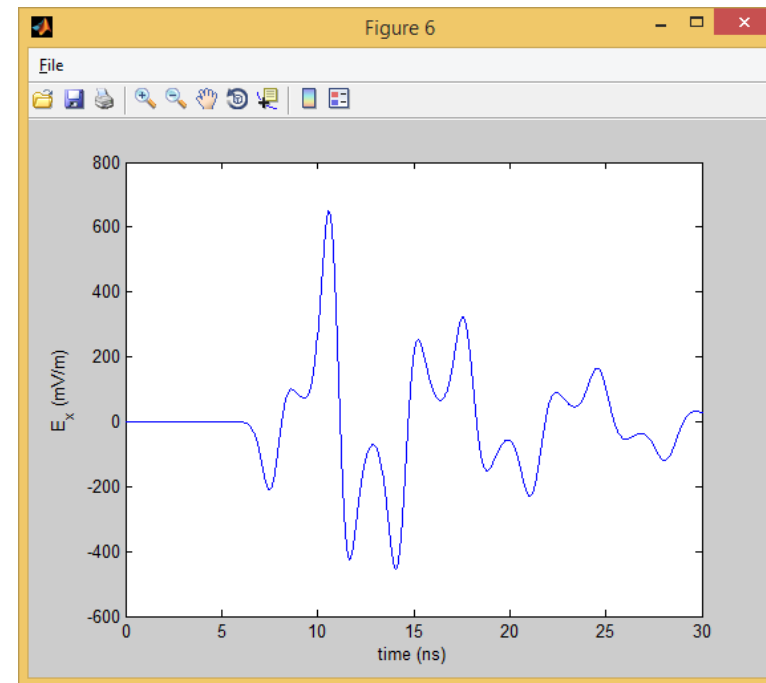
Time Domain (TD)

Inverse Fast Fourier Transform (IFFT) to TD

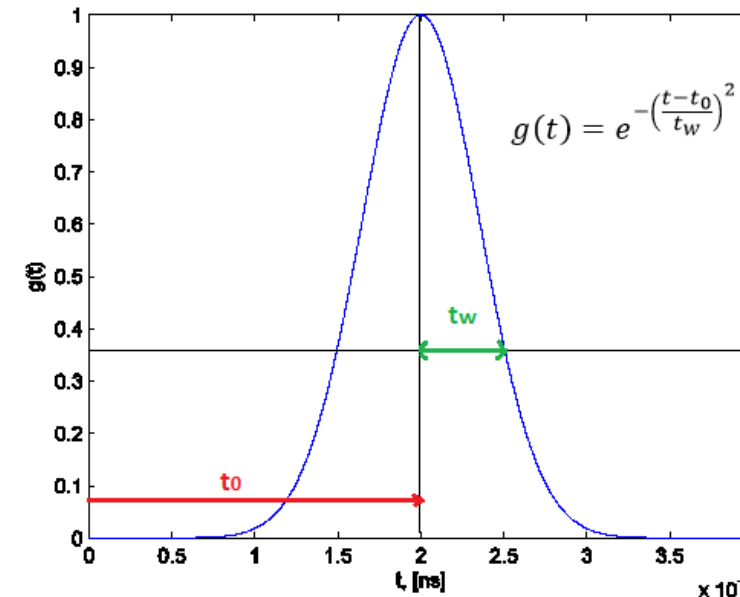
time delay (ns)	2	time frame (ns)	40
halwidth (ns)	0.667	x (m)	0.5
		z (m)	0.5

Show Gauss pulse

Save frequency domain field



- The excitation is given as a Gaussian waveform where the parameters  $t_w$  and  $t_0$  represent the half width and time delay of a pulse
- Figure generated upon the calculation:
  - E vs. time for tangential component of electric field



# Examples

1. E vs. xz
2. E vs. z
3. E vs. freq
4. E vs. time



# 1. E vs. xz

Field transmitted into ground: frequency domain integral equation approach

Antenna parameters  
length: 1, radius (mm): 6.74, height (cm): 10

Ground parameters  
Layer 1, Layer 2, **Layer 3**  
permittivity: 10, 8, 10  
conductivity (mS/m): 0, 0, 0  
thickness (m): 0.25, 0.3, [ ]

Run the simulation  
Method: **MIT**, RCA  
Run the simulation  
Results are saved in: [ ]  
FD: [ ]  
TD: [ ]

Frequency domain (FD)  
**Single frequency**, Single point, xz grid, multiple F  
Coordinates (m):  
x axis: START -0.5, END 1.5, No. points 50  
z axis: START 0, END 2, No. points 50  
Frequency (MHz): **10**, 150, 15

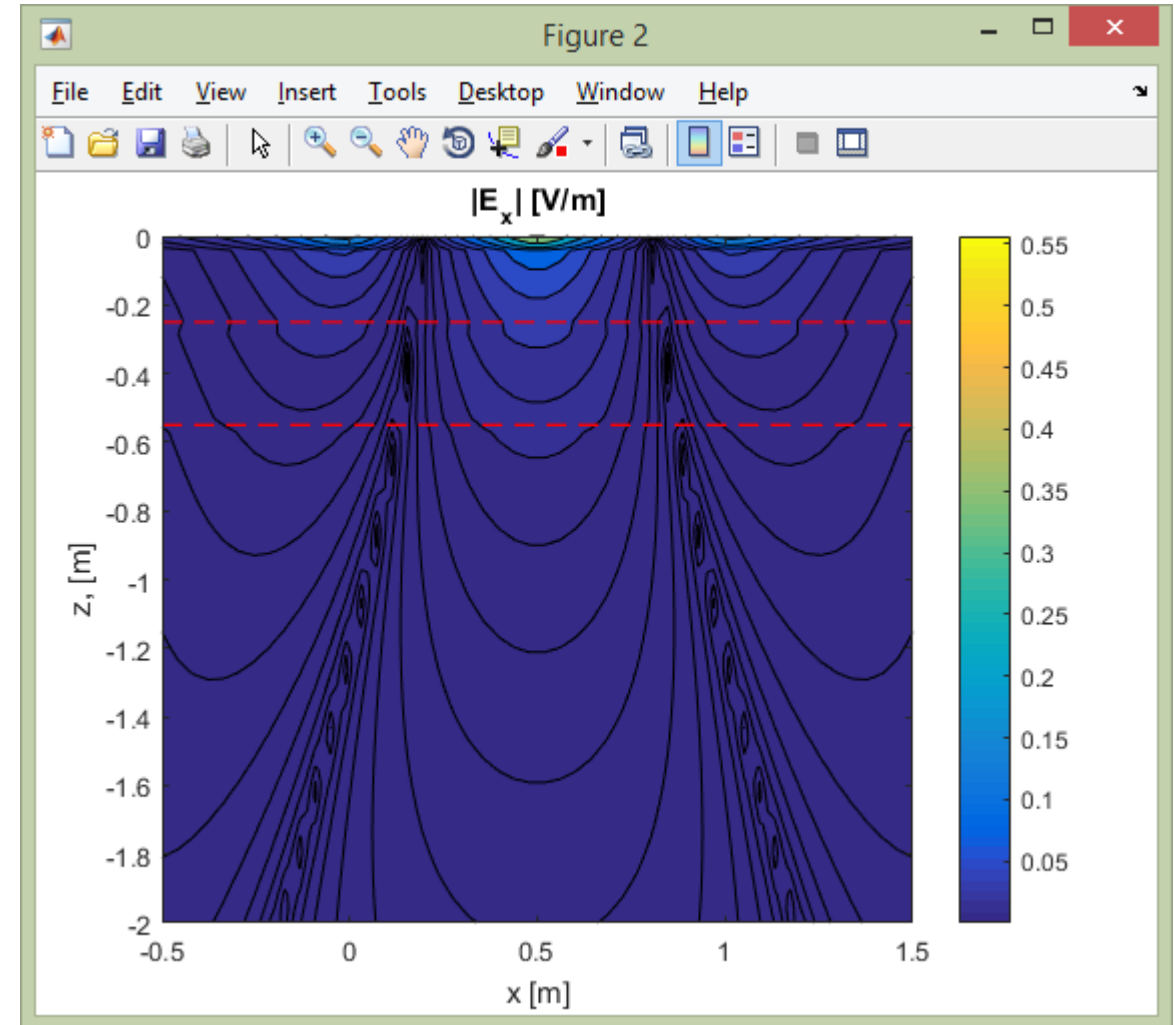
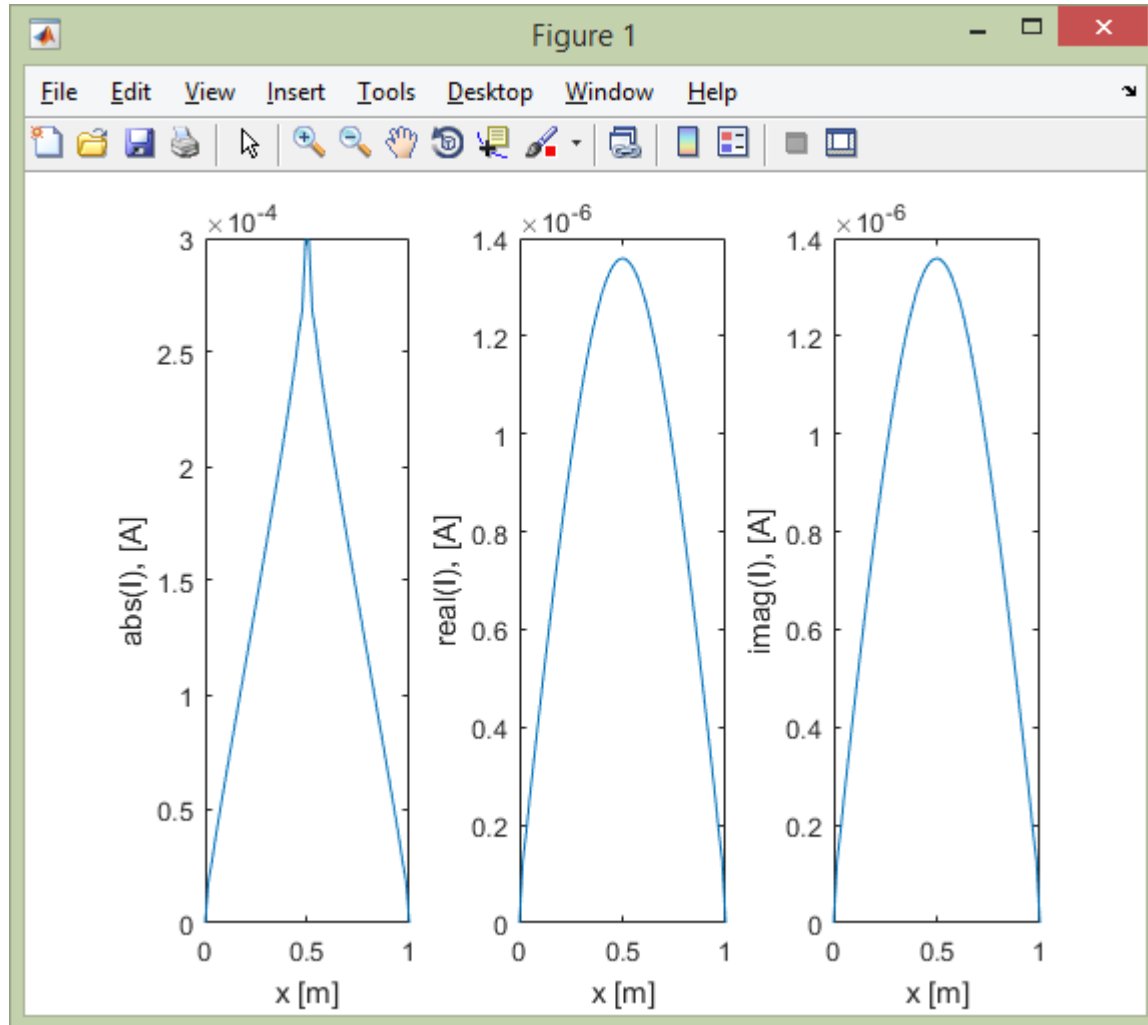
Time Domain (TD)  
Inverse Fast Fourier Transform (IFFT) to TD  
time delay (ns): 1.43, time frame (ns): 40  
halwidth (ns): 0.667, x (m): 0.5, z (m): 0.5  
Show Gauss pulse  
Save frequency domain field [ ]

Single frequency.  
Frequency = 10.00 MHz  
[Progress bar]  
Cancel

# 1. E vs. xz

$f = 10 \text{ MHz}$

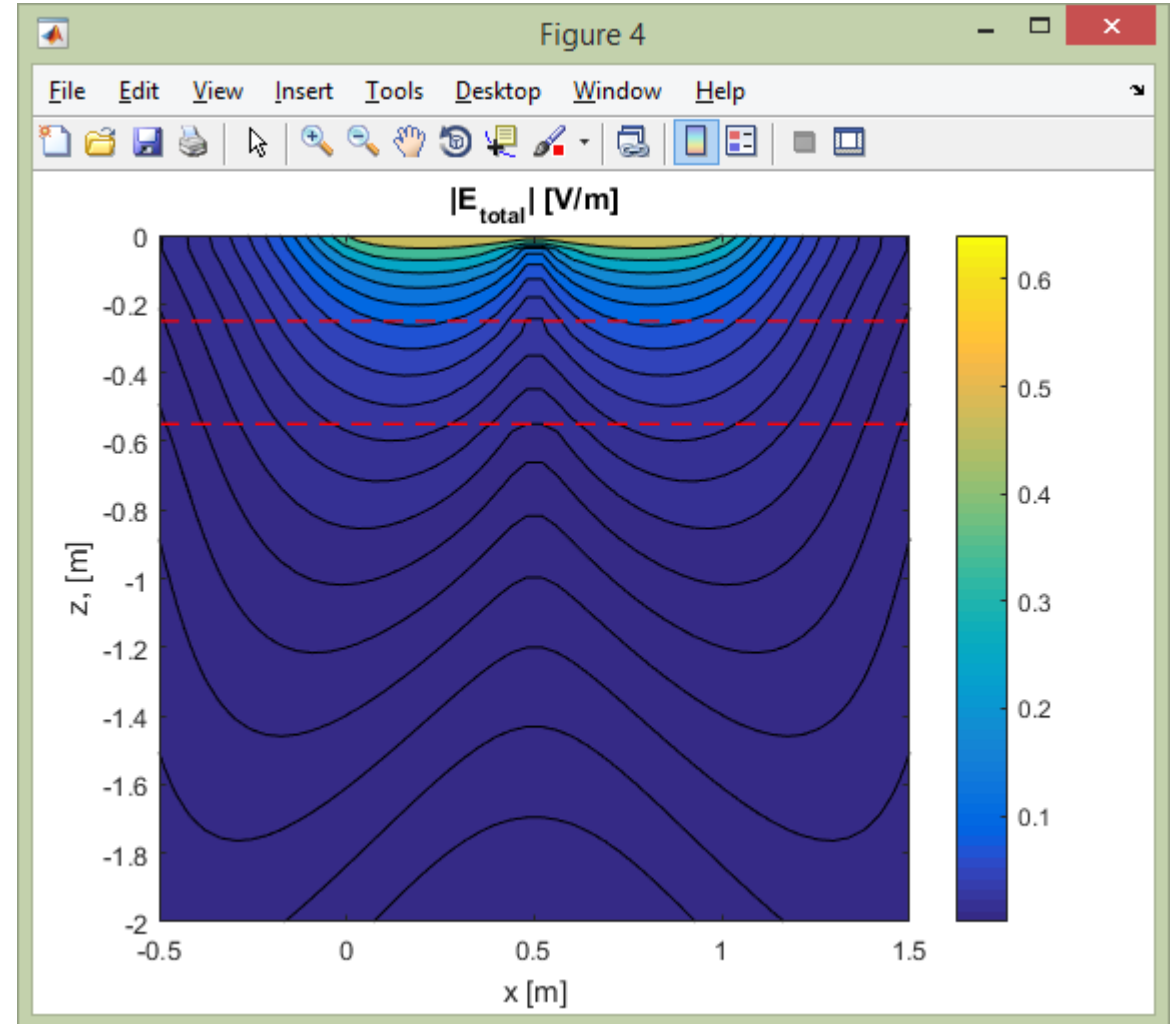
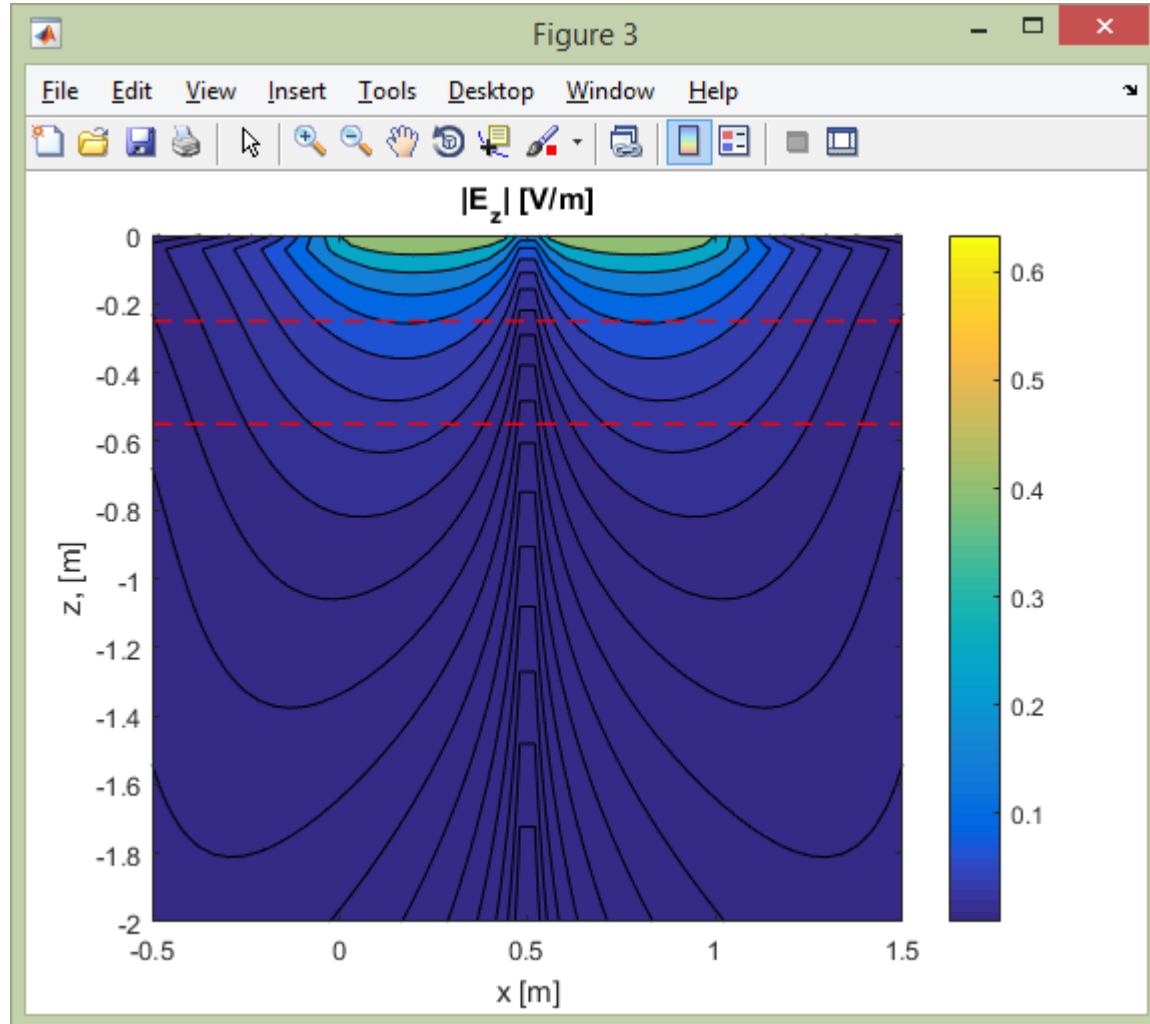
26



# 1. E vs. xz

$f = 10 \text{ MHz}$

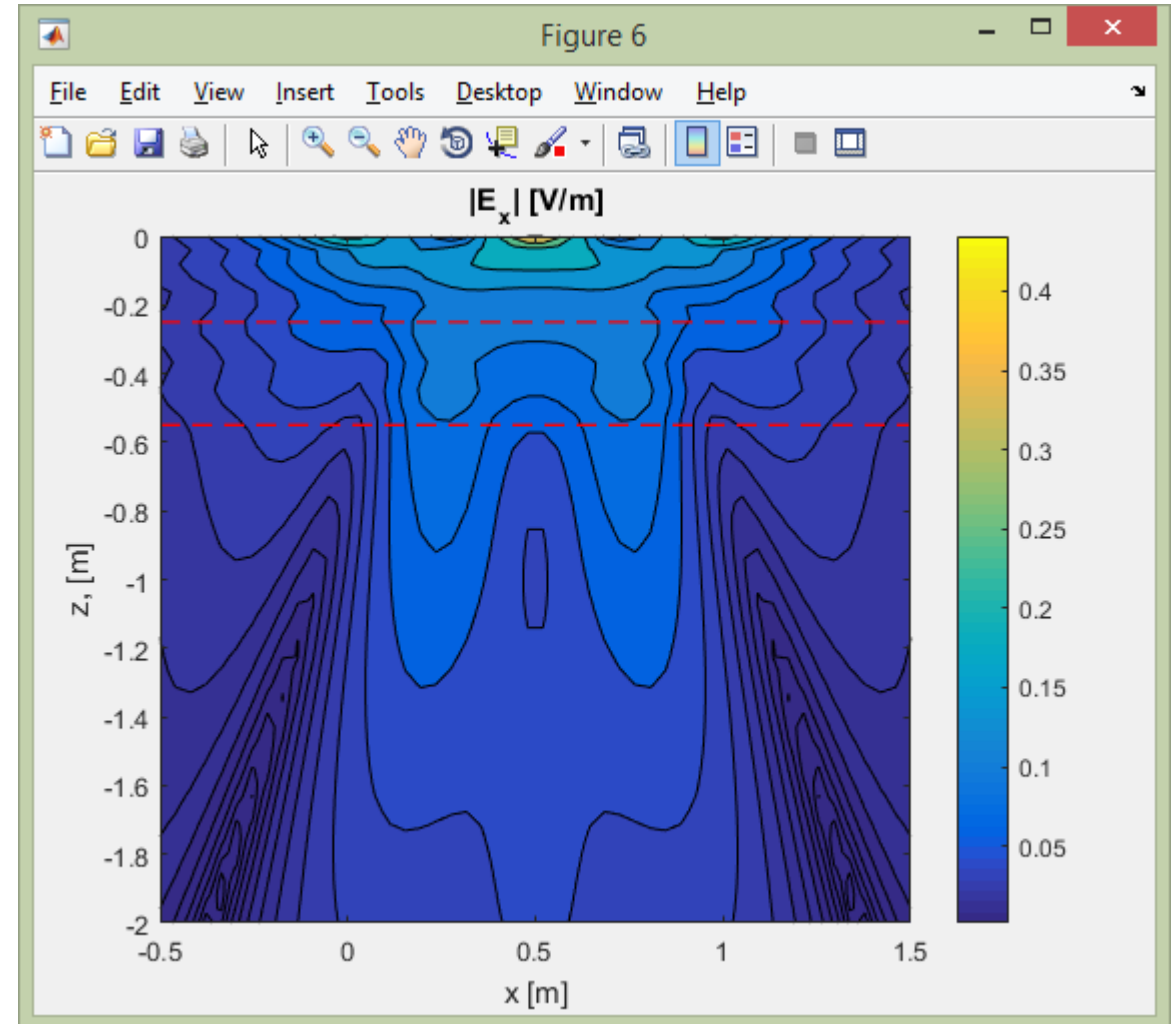
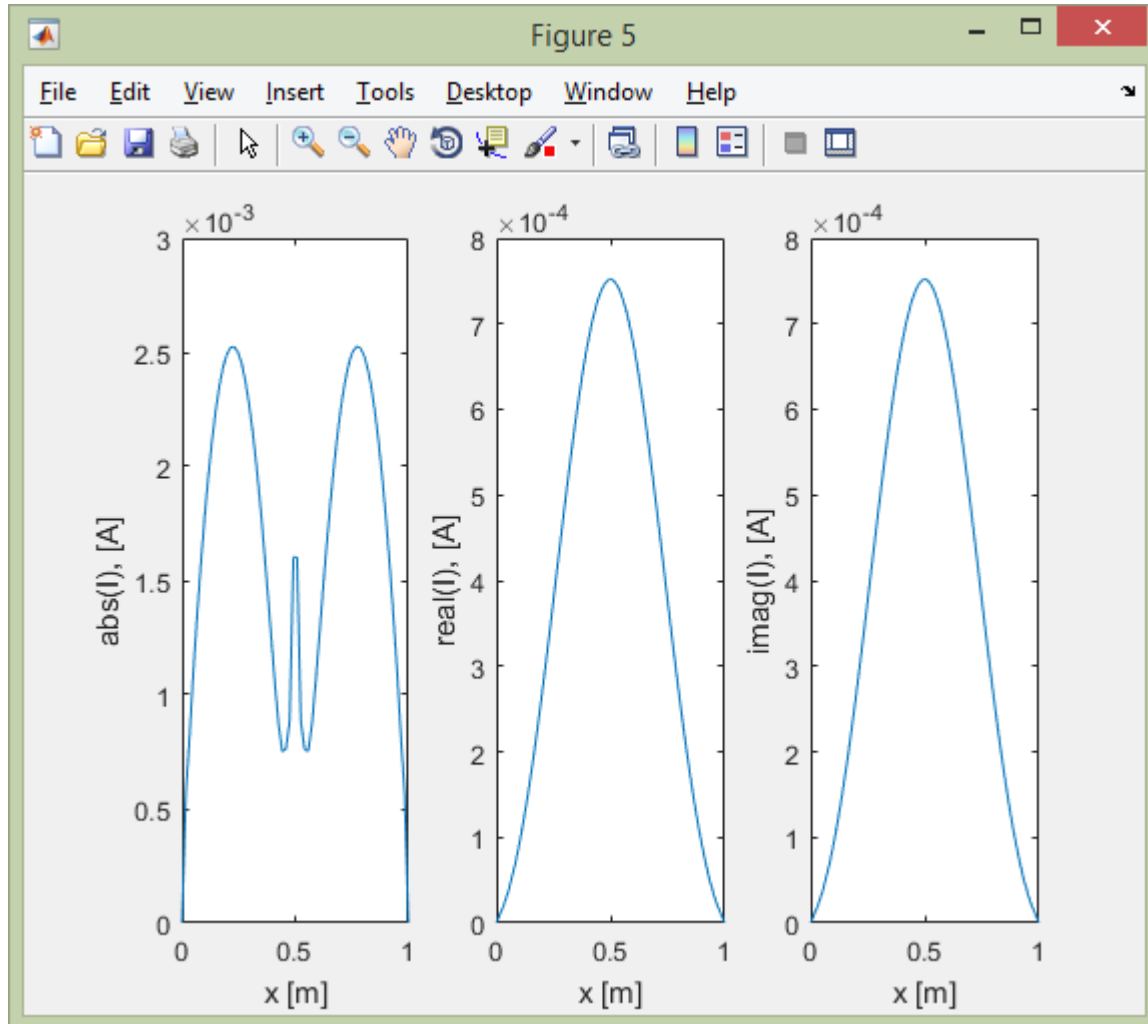
27



# 1. E vs. xz

$f = 300 \text{ MHz}$

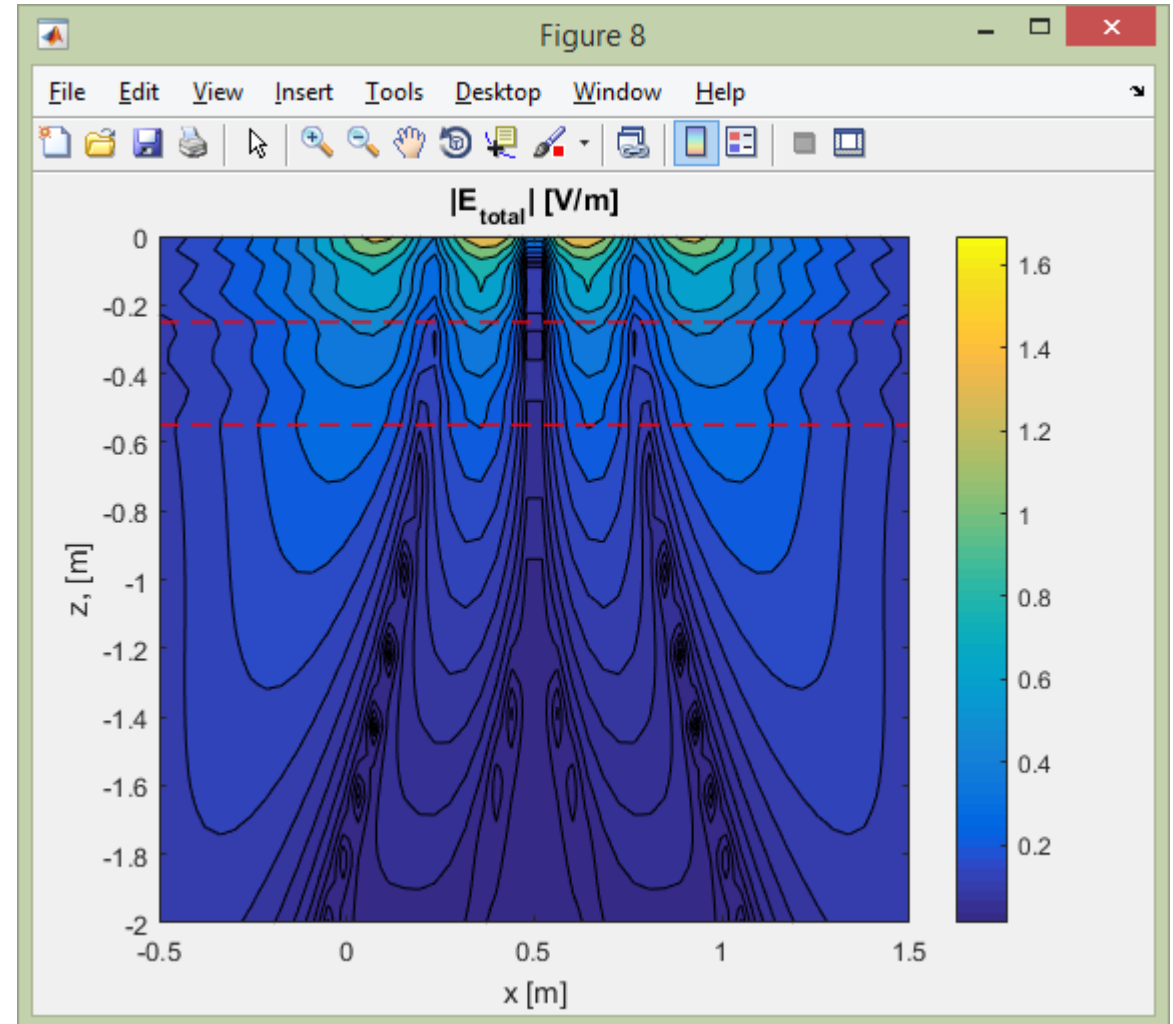
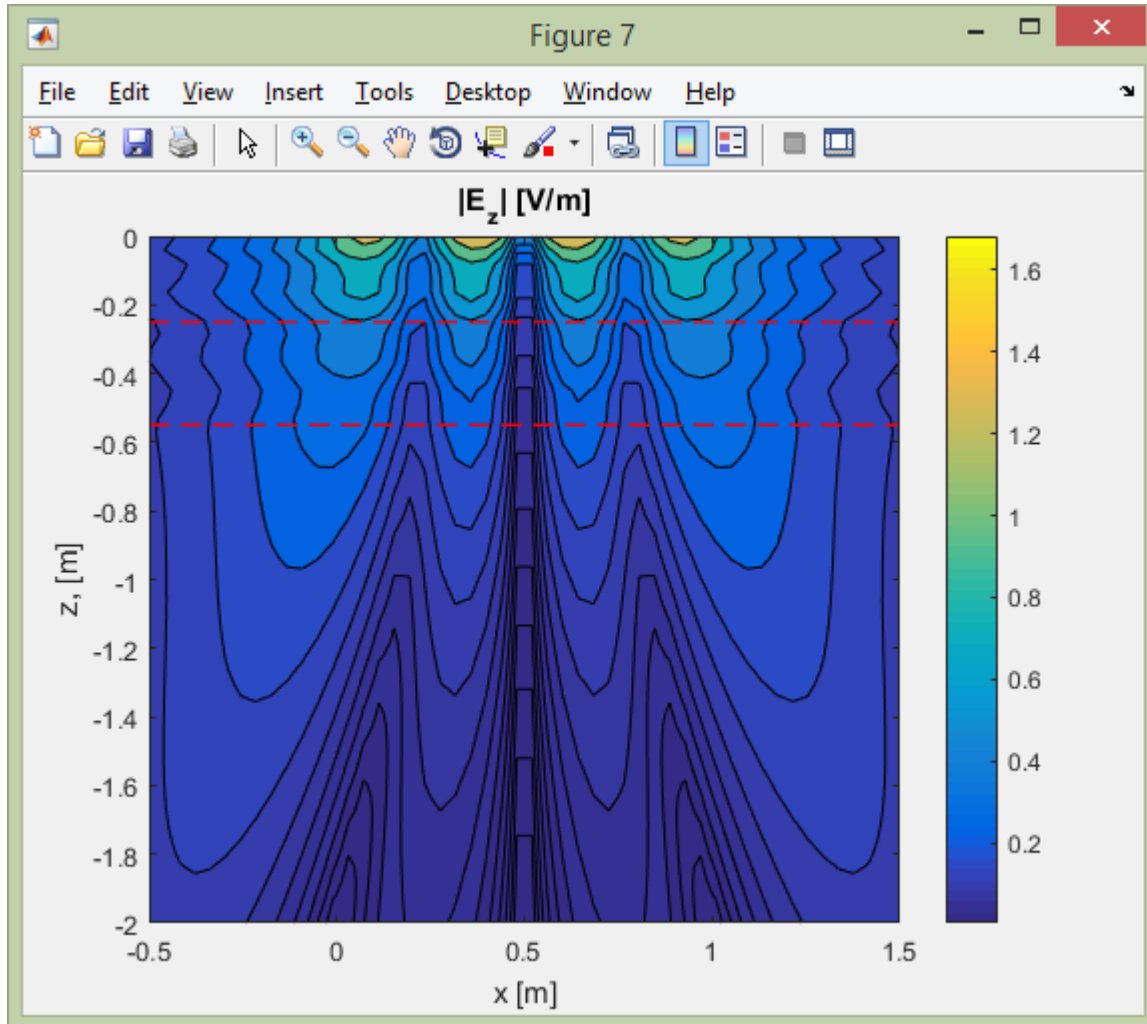
28



# 1. E vs. xz

$f = 300 \text{ MHz}$

29



# 1. E vs. XZ

30

The image shows a Windows file explorer window titled "Save as" with the path "This PC > Desktop > test". The file name is "E\_vs\_xz" and the save type is "All Files". An inset window shows a file list with two files: "E\_vs\_xz\_CURRENT" (4 KB) and "E\_vs\_xz\_FIELD" (279 KB), both dated 14.5.2018. 10:39. A third inset window titled "Run the simulation" shows the "Method" set to "RCA" and the "Results are saved in:" field containing "C:\Users\lansus\_000\Desktop\test\E\_vs\_xz".

Name	Date modified	Type	Size
E_vs_xz_CURRENT	14.5.2018. 10:39	Text Document	4 KB
E_vs_xz_FIELD	14.5.2018. 10:39	Text Document	279 KB

Run the simulation

Method:  MIT  RCA

Run the simulation

Results are saved in: Save

FD: C:\Users\lansus\_000\Desktop\test\E\_vs\_xz

TD:

Save Cancel

# 1. E vs. xz

Freq. (MHz)            x(m)            Real(I)            Imag(I)

Freq. (MHz)	x(m)	Real(I)	Imag(I)
300000000.000000	0.000000	0.000000	0.000000
300000000.000000	0.016393	0.000011	-0.000570
300000000.000000	0.032787	0.000021	-0.000814
300000000.000000	0.049180	0.000035	-0.001096
300000000.000000	0.065574	0.000050	-0.001337
300000000.000000	0.081967	0.000069	-0.001562
300000000.000000	0.098361	0.000090	-0.001764

Freq. (MHz)            x(m)            z(m)            Real( $E_x$ )            Imag( $E_x$ )            Real( $E_z$ )            Imag( $E_z$ )

Freq. (MHz)	x(m)	z(m)	Real( $E_x$ )	Imag( $E_x$ )	Real( $E_z$ )	Imag( $E_z$ )
300000000.000000	-0.500000	0.000000	-0.028258	0.002993	0.093729	-0.101466
300000000.000000	-0.459184	0.000000	-0.029324	-0.012547	0.145006	-0.043515
300000000.000000	-0.418367	0.000000	-0.022069	-0.028614	0.161904	0.040404
300000000.000000	-0.377551	0.000000	-0.006098	-0.040939	0.131095	0.130706
300000000.000000	-0.336735	0.000000	0.016947	-0.044938	0.049877	0.200821
300000000.000000	-0.295918	0.000000	0.043190	-0.036573	-0.070137	0.222559
300000000.000000	-0.255102	0.000000	0.066621	-0.013279	-0.201731	0.173307
300000000.000000	-0.214286	0.000000	0.079468	0.025125	-0.303865	0.044179
300000000.000000	-0.173469	0.000000	0.072807	0.075483	-0.326867	-0.151681
300000000.000000	-0.132653	0.000000	0.037764	0.130120	-0.221772	-0.370922
300000000.000000	-0.091837	0.000000	-0.030657	0.175601	0.042623	-0.533297

Field transmitted into ground: frequency domain integral equation approach

**Antenna parameters**  
 length: 1, radius (mm): 6.74, height (cm): 10

**Ground parameters**  
 Layer 1: permittivity 10, conductivity 0, thickness 0.25  
 Layer 2: permittivity 8, conductivity 0, thickness 0.3  
 Layer 3: permittivity 10, conductivity 0, thickness (empty)

**Frequency domain (FD)**  
 Single frequency,  Single point,  xz grid, multiple F

Coordinates (m)	START	END	No. points
x axis	0.5	0.5	1
z axis	0	2	50
Frequency (MHz)	100	150	

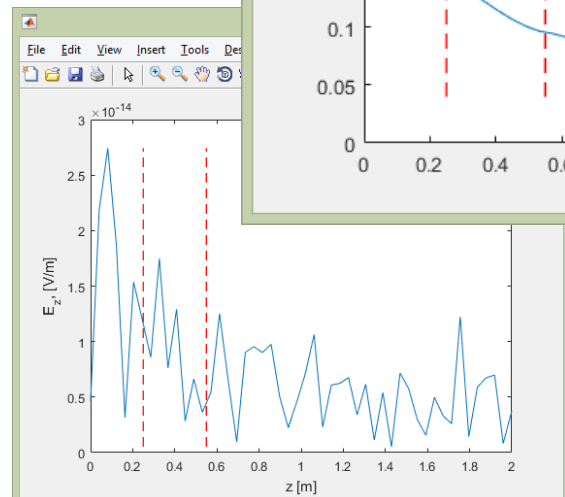
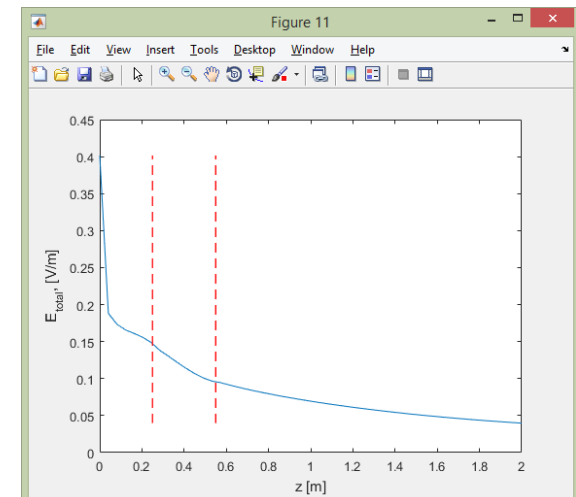
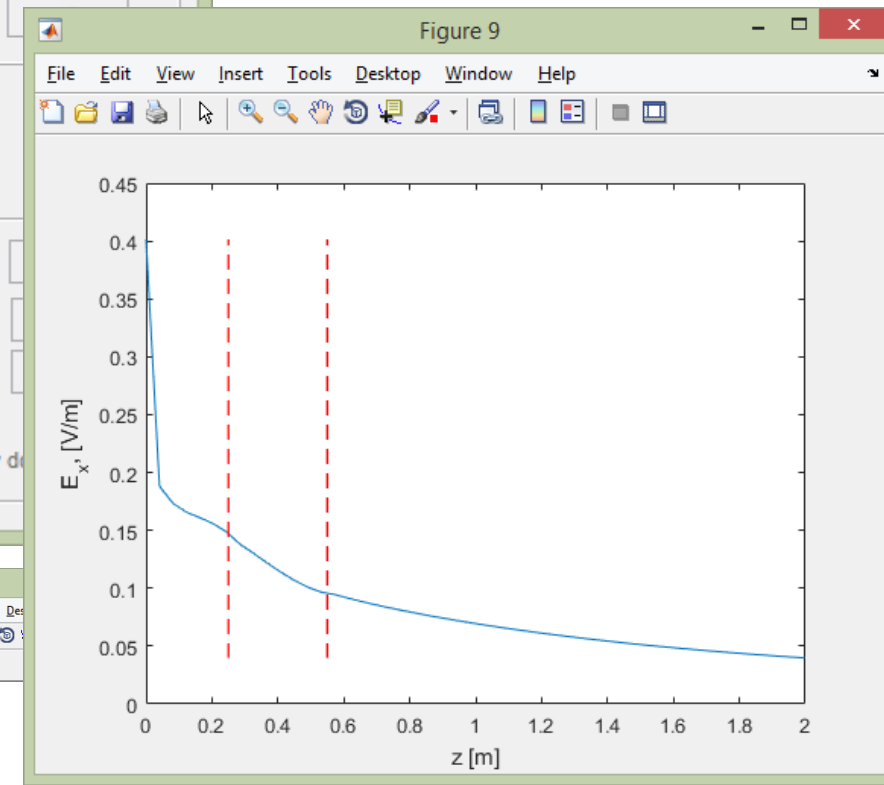
**Time Domain (TD)**  
 Inverse Fast Fourier Transform (IFFT) to TD

time delay (ns): 1.43, time frame (ns):  
 halwidth (ns): 0.667

Method:  MIT,  RCA

Run the simulation, Save

Results are saved in:  
 FD:   
 TD:



## 2. E vs. z



Field transmitted into ground: frequency domain integral equation approach

**Antenna parameters**

length  radius (mm)  height (cm)

**Ground parameters**

Layer 1  Layer 2  Layer 3

permittivity

conductivity (mS/m)

thickness (m)

**Run the simulation**

Method:  MIT  RCA

Results are saved in:

FD:

TD:

**Frequency domain (FD)**

Single frequency  Single point  xz grid, multiple F

Coordinates (m)	START	END	No. points
x axis	<input type="text" value="0.5"/>	<input type="text" value="0.5"/>	<input type="text" value="1"/>
z axis	<input type="text" value="0"/>	<input type="text" value="2"/>	<input type="text" value="50"/>
Frequency (MHz)	<input type="text" value="100"/>	<input type="text" value="150"/>	<input type="text" value="15"/>

**Time Domain (TD)**

Inverse Fast Fourier Transform (IFFT) to TD

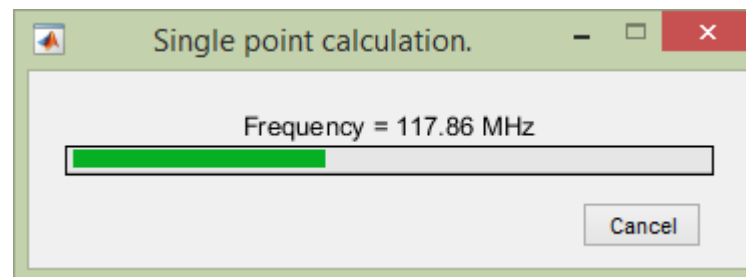
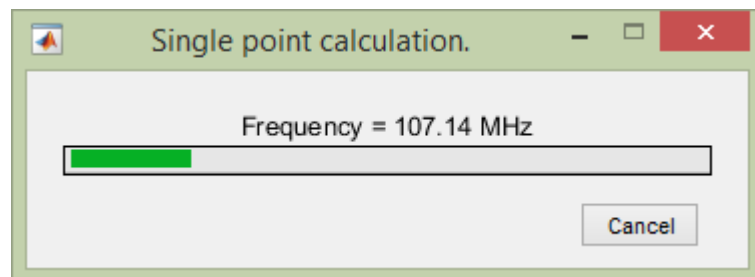
time delay (ns)  time frame (ns)

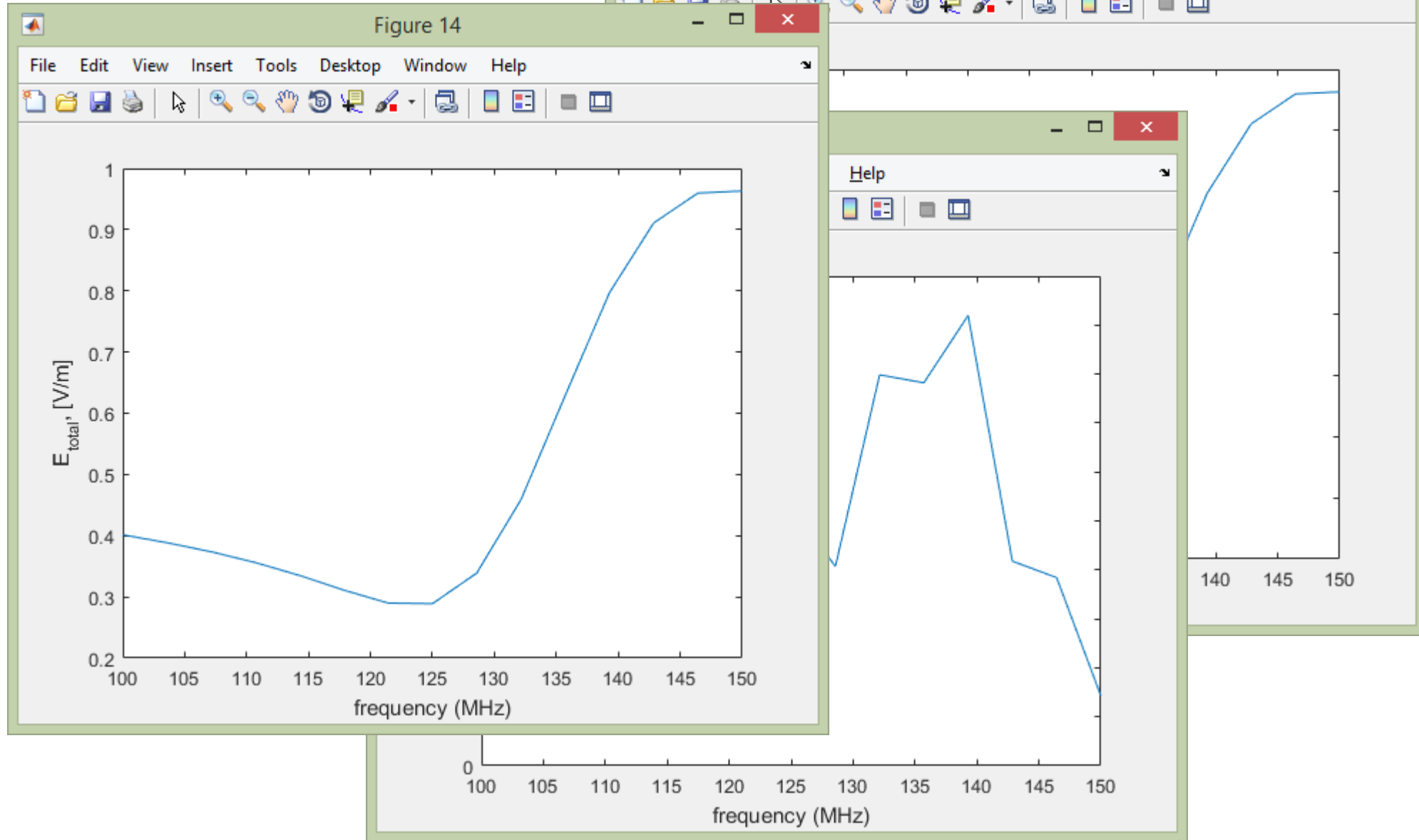
halwidth (ns)  x (m)

z (m)

Save frequency domain field

## 3. E vs. f





### 3. E vs. f

### 3. E vs. time

Field transmitted into ground: frequency domain integral equation approach

**Antenna parameters**  
length: 1    radius (mm): 6.74    height (cm): 10

**Ground parameters**  
Layer 1: permittivity 10, conductivity 0, thickness 0.25  
Layer 2: permittivity 8, conductivity 0, thickness 0.3  
Layer 3: permittivity 10, conductivity 0, thickness (empty)

**Frequency domain (FD)**  
 Single frequency     Single point     xz grid, multiple F

Coordinates (m)	START	END	No. points
x axis	0.5	0.5	1
z axis	0	2	50
Frequency (MHz)	100	150	15

**Time Domain (TD)**  
 Inverse Fast Fourier Transform (IFFT) to TD

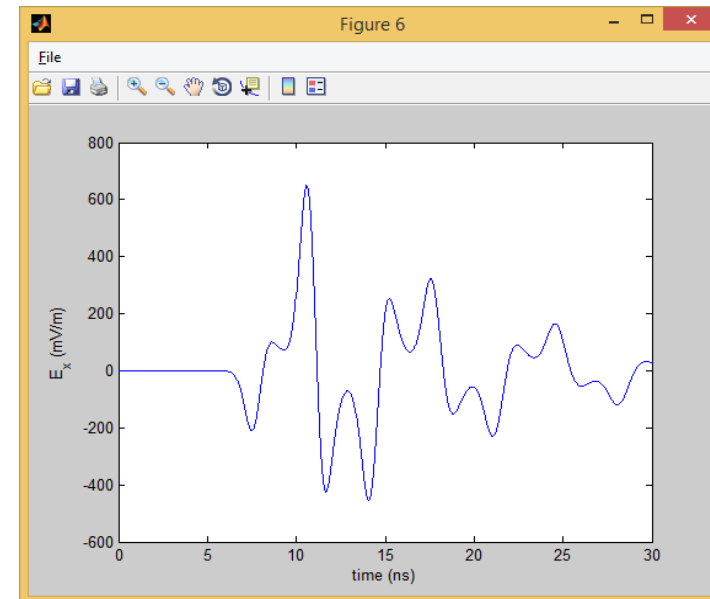
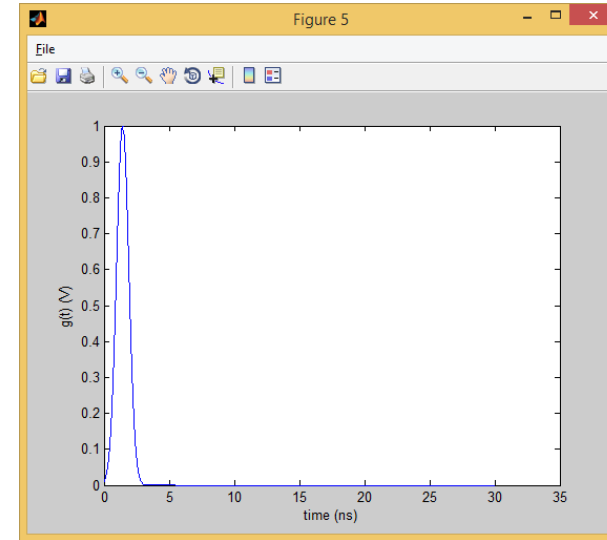
time delay (ns): 1.43    time frame (ns): 40  
halwidth (ns): 0.667    x (m): 0.5  
z (m): 0.2

Save frequency domain field

Method:  MIT     RCA    Run the simulation

Results are saved in: Save

FD:   
TD:



## REFERENCES

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- [3] A. Šušnjara, D. Poljak, S. Šesnić, V. Dorić “Time Domain Integral Equation versus Frequency Domain Boundary Element model for calculation of Transmitted Electrical Field for Ground Penetrating Radar (GPR) Antenna
- [4] A. Šušnjara, D. Poljak, V. Dorić, “Electric Field Radiated By a Dipole Antenna Above a Lossy Half Space: Comparison of Plane Wave Approximation with the Modified Image Theory Approach,” SoftCOM conference, Split, 2017
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